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Multicriteria Optimization of Cellular Networks

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ABSTRACT

When designing modern cellular networks, it is challenging to account for many contradictory criteria and constantly changing external conditions of the networks (e.g., traffic). We need to solve multicriteria problems with high-dimensional vectors of parameters. A prerequisite to solution of these problems is correct determination of the feasible solution set, which is directly related to the statement of optimization problem. This is a major challenge in all multicriteria engineering optimization problems and represents significant difficulties for the expert. In this paper, we show how to define the feasible solution set for cellular network optimal design problems and thus answer the fundamental question of where to search for optimal solutions in such problems. We use the Parameter Space Investigation (PSI) method implemented in the Multicriteria Optimization and Vector Identification (MOVI) software system and apply it to a mathematical model of cellular network. In addition to developing methodology for stating and solving the problem of multicriteria optimization of cellular network, we have found that 1) defining the feasible solution set is directly related to the correct statement of the optimization problem, 2) once the feasible solution set has been determined, the criteria convolution can be applied to find the optimal solution in the feasible solution set, 3) it is possible to perform online tuning of the cellular network parameters.

Keywords: Feasible Solution Set; Pareto Optimal Solutions; Parameter Space Investigation (PSI) Method; Cellular Networks; Multicriteria Problems

1. Introduction

One of the distinguishing characteristics of cellular networks is difficulty of their design given a large number of contradictory criteria and constantly changing external conditions of the networks (e.g., traffic). In addition to multiple criteria, we deal with vectors of parameters of high dimensionality (hundreds to thousands). Optimal design of cellular networks is at the interface of design and management problems, and there have hardly been any attempts to construct a feasible solution set for problems of such type. The feasible solution set is essential because it contains a Pareto set of optimal solutions, none of which can be improved along all quality criteria simultaneously. In addition to the difficulties in determining the feasible solution set, choice of the best solution from a Pareto set is also non-trivial when this set contains a large number of solutions. In some cases, these difficulties can be mitigated by applying criteria convolution as described below. However, in order to apply convolutions correctly, one has to define feasibility of obtained solution first, which makes a proper construction of the feasible solution set the key factor. In this work, we consider applying the Pareto Space Investigation (PSI) method as an attempt to improve existing techniques of network design and management by constructing and analyzing the feasible solution set. The goal of this paper is to show feasibility and capabilities of the PSI method for solving problems of optimal design of cellular networks.

Most existing works on cellular network optimization that deal with multi-criteria problems transform them into single-objective variant using some scalarization techniques [1,2], weighted sum being the most popular approach. Melachrinoudis and Rosyidi [3] use simulated annealing algorithm to optimize weighted sum of three criteria (call quality, demand coverage and total cost) by varying location, power and antenna height of base stations. Galota *et al*. [4] choose positions of base stations to optimize weighted sum of three criteria (number of supplied demand nodes, ongoing costs, and cost of intra-cell interference) subject to constraint on construction costs. Amaldi *et al*. [5] aim at maximizing total covered traffic and minimizing total installation costs by forming weighted sum of these criteria; two-stage Tabu Search algorithm is used starting from solution provided by randomized greedy procedure. Hurley [6] proposes framework based on Simulated Annealing for optimization of objective function that is sum of five criteria (coverage, site cost, traffic, interference, and handover). Gerdenitsch [7] uses sum of number of served users, coverage, and soft handover as objective function for the problem of tuning transmit power and antenna downtilt using Genetic Algorithm. Zhu and Buot [8] propose heuristic algorithm for online network optimization based on estimating linear model between KPIs (key performance indicators) and parameters (expressed in form on sensitivity matrix) from a-priori simulation results and KPI measurements. Weighted sum of five KPIs is used as optimization process performance index. The approach allows specifying KPI targets.

In few works ε-constraint technique is utilized: one of criteria is selected to be optimized, and others are converted into constraints [1,2]. One of network optimization problems studied by Siomina [9] is example of such approach: total network load (total pilot power) is minimized subject to coverage constraint. Other approaches not based on some form of scalarization (weighted sum or ε-constraint) are much less common. Jedidi *et al*. [10] argue that it makes sense to consider cells overlap and cells geometry as criteria for real-life network optimization. They aim at finding the whole Pareto front of this bi-criteria problem using a version of Multiobjective Evolutionary Algorithm. It is well known that such algorithms are quite effective for two or three criteria problems, as higher number of criteria their applicability is an open question [11]. To the best of our knowledge, most commercial cellular network planning and optimization tools are also based on some form of scalarization such as weighted sum [12,13].

2. Problem Formulation

In this paper, we are concerned with multicriteria optimization problems arising during planning and operation of cellular mobile network. Mobile network provides service in area divided into cells, each served by transceiver of base station that has several tunable parameters affecting network performance like transmit power, antenna orientation, associated radio frequency, parameters of radio resource management algorithms, etc. Typically these parameters are configured by experts (possibly with the help of network planning and optimization tools) during network planning stage and kept fixed for a long period of time. Cellular network demand and environment are ever changing during day, and such fixed configuration (usually targeted at peak network load-busy hour) can become not well suited for current network conditions. It is also possible to change some of cellular network parameters online trying to adapt to changing conditions. Thus network optimization can be performed in offline (with full set of parameters available for configuration and lots of time for decision making) or online (with restricted set of parameters available and limited time for decision making) mode. We propose using of the Parameter Space Investigation method in both of the network optimization modes.

Reference signal transmit power and antenna electrical downtilt are one of the most important parameters that have great impact on different network KPIs such as capacity and coverage. Moreover, these two parameters can be changed remotely and automatically as opposed to manual and costly reconfiguration of some other important parameters (such as antenna azimuth or mechanical downtilt). This is the motivation for using the above two parameters for our study.

Whole network area is divided into pixels using some grid (typical size is 50 m × 50 m), each pixel is characterized by receive power level from each cell in network and mean number of users in it. The former aggregates into signal propagation maps (one for each cell), and the latter into traffic map (traffic maps can be differentiated by type of service). Reference signal received power is reference signal transmit power times channel gain between cell and pixel, influenced by antenna downtilt of this cell. Increasing transmit powers of all cells and pointing antennas up (corresponds to small downtilt values), we increase coverage in network (received power in each pixel becomes high enough to connect to the cell with strongest signal) but at the same time we increase interference between cells, which effectively leads to network capacity degradation. Trade-off between these objectives is not obvious, and needs to be carefully studied. It is hard to say beforehand what KPI values are achievable [12-15].

2.1. System Model

Let us denote by $c = 1, \ldots, C$ the set of cells serving planning area represented by a grid of pixels. Signal propagation (averaged in time and frequency) from each cell c to each pixel m is characterized by power gain $g_{mc}A_{mc}$, composed of isotropic gain g_{mc} and antenna gain A_{mc}. Antenna gain A_{mc} depends on electrical downtilt t_c of antenna associated with cell c and direction (given by azimuth ϕ_{mc} and elevation θ_{mc} angles) between this antenna and pixel m: $A_{mc} = G_c H_c(t_c, \phi_{mc}) V_c(t_c, \theta_{mc})$, where G_c is cell's c antenna maximum gain, $H_c(\cdot,\cdot)$ and $V_c(\cdot,\cdot)$ are tabulated functions of horizontal and vertical cell c antenna patterns [14].

Reference Signal Received Power (RSRP) R_{mc} from cell c in pixel m is calculated as

$$R_{mc} = P_c g_{mc} A_{mc},$$

where P_c is reference signal transmit power of cell c.

Each pixel m is associated with cell c with largest received power R_{mc} among all other cells, for such pixel–cell pair m, c we set association indicator a_{mc} equal to 1. Signals from all other cells interfere with useful signal from serving cell. One of the most important link quality indicators is Reference Signal Signal-to- Interference and Noise Ratio (RSSINR) given by

$$\gamma_{mc} = \frac{R_{mc}}{\sigma^2 + \sum_{d \in \mathcal{C} \setminus \{c\}} R_{md}},$$

where σ^2 is noise power.

Each cell has frequency resources (resource blocks in LTE) that it allocates between served users. We use definition of cell load as long-term average percentage of utilized frequency resources [16]. Cell load depends on amount of traffic requested by served users and their link capacities. We characterize long-term average requested traffic in pixel m using T_m—average number of users in this pixel requesting some service, and D—bitrate demand of this service. Load of cell c required to serve its users is estimated as

$$\rho_c = D \sum_m \frac{a_{mc} T_m}{W_c \log_2 \left(1 + \gamma_{mc}\right)},$$

where W_c is bandwidth of frequency resources available in cell c. Here we used Shannon formula for link capacities.

2.2. Performance Criteria

Call Drop and/or Block Rate (CDBR) performance criterion Φ_{CDBR} is determined by cell loads and number of served users as

$$\Phi_1 = \Phi_{\text{CDBR}} = \frac{\sum_c \max\left(0, 1 - 1/\rho_c\right) \sum_m a_{mc}}{\sum_c \sum_m a_{mc}}.$$

Percentage of RSRP and RSSINR covered area criteria $\Phi_{\text{RSRP-covA}}$ and $\Phi_{\text{RSSINR-covA}}$ are defined as

$$\Phi_2 = \Phi_{\text{RSRP-covA}} = \frac{\sum_c \sum_m a_{mc} h\left(R_{mc}, R^0\right)}{\sum_c \sum_m a_{mc}}$$

and

$$\Phi_3 = \Phi_{\text{RSSINR-covA}} = \frac{\sum_c \sum_m a_{mc} h\left(\gamma_{mc}, \gamma^0\right)}{\sum_c \sum_m a_{mc}},$$

where R^0 and γ^0 are RSRP and RSSINR thresholds and $h\left(x, x^0\right) = \begin{cases} 1 & x \ge x^0 \\ 0 & \text{otherwise} \end{cases}$. In a similar way percentage of RSRP and RSSINR covered traffic criteria criteria $\Phi_{\text{RSRP-covT}}$ and $\Phi_{\text{RSSINR-covT}}$ are defined as

$$\Phi_4 = \Phi_{\text{RSRP-covT}} = \frac{\sum_c \sum_m a_{mc} T_{mc} h\left(R_{mc}, R^0\right)}{\sum_c \sum_m a_{mc} T_{mc}}$$

and

$$\Phi_5 = \Phi_{\text{RSSINR-covT}} = \frac{\sum_c \sum_m a_{mc} T_m h\left(\gamma_{mc}, \gamma^0\right)}{\sum_c \sum_m a_{mc} T_m}.$$

Mean SINR (Signal to Interference and Noise Ratio) [dB] KPI is defined as

$$\Phi_6 = \Phi_{\text{M-SINR}} = \frac{10}{\sum_m T_m} \sum_c \sum_m a_{mc} T_m \log_{10}\left(\gamma_{mc}\right).$$

Mean required cell load criterion is

$$\Phi_7 = \Phi_{\text{M-load}} = \frac{\sum_c \rho_c}{C}.$$

Aggregated criterion is weighted sum of the above seven criteria $\Phi_{\text{aggr}} = \sum_{i=1}^{7} \beta_i \Phi_i$. Usually cellular network operator has well-defined targets that should be achieved for RSRP-covA, RSRP-covT, and CDBR criteria, and additionally we would like to maximize/minimize the following criteria shown in **Table 1**.

In our examples, we consider network consisting of 50 cells with 100 variables. Variables are transmit power p_c and electrical downtilt t_c, c = 1, ..., 50. The considered network with traffic map is shown in **Figure 1**. In the

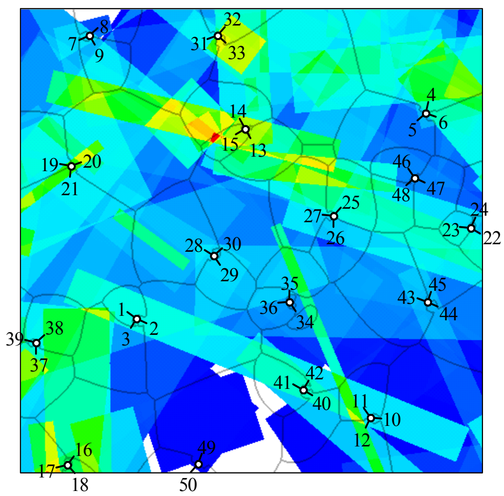

Figure 1. Map of cellular network used in examples. Color intensity corresponds to traffic map (ranging from blue–small number of users, to red–large). Base stations are located at white dots, each having three antennas (one antenna serves one cell). Short black line segments represent horizontal direction of corresponding antennas. Numbers are cell indices. Grey lines are cell borders. For example cells with indices 10, 11, and 12 are served by transceivers of one base station, antenna of cell 10 is directed towards East, 11—Northwest, and 12—Southwest.

Table 1. Criteria names, meaning and objective.

Criterion	Name	Meaning	Objective
Φ_1	CDBR	Call Drop and/or Block Rate	min
Φ_2	RSRP-covA	Percentage of RSRP (Reference Signal Received Power) covered area	max
Φ_3	RSSINR-covA	Percentage of RSSINR (Reference Signal Signal-to-Interference and Noise Ratio) covered area	max
Φ_4	RSRP-covT	Percentage of RSRP covered traffic	max
Φ_5	RSSINR-covT	Percentage of RSSINR covered traffic	max
Φ_6	Mean-SINR	Mean SINR (Signal to Interference and Noise Ratio) [dB]	max
Φ_7	M-load	Mean required cell load	min

first problem (Task 1), $0 \le p_c \le 40$ W, $0^\circ \le t_c \le 10^\circ$ of each cell, $c = 1, ..., 50$. In the second problem (Task 2), $0 \le p_c \le 50$ W. The ranges of change for the electrical downtilt are kept unchanged by $0^\circ \le t_c \le 10^\circ$ of each cell, $c = 1, ..., 50$. Electrical downtilt constraints are determined by capabilities of antennas serving each cell. Transmit power constraints are also due to hardware limitations (power amplifier) and electromagnetic radiation regulations.

In what follows we will show how to construct the feasible solution set for the above set of performance criteria.

3. Construction of the Feasible and Pareto Optimal Solution Sets

3.1. Generalized Formulation of Multicriteria Optimization Problems

We discuss here a mathematical formulation that can be applied to the majority of engineering optimization problems [17-23]. In the general case, when designing a system, one has to take into account the design variable constraints, the functional constraints, and the criteria constraints.

The *design variable constraints* (constraints on the design variables) have the form:

$$\alpha_j^* \le \alpha_j \le \alpha_j^{**},\ j = 1,\ldots,r. \quad (1)$$

The *functional constraints* on *functional dependences* $f_l(\alpha)$ may be written as follows:

$$C_l^* \le f_l(\alpha) \le C_l^{**}, l = 1,\ldots,t, \quad (2)$$

where C_l^* and C_l^{**} are, respectively, the lower and the upper admissible values of the quantity $f_l(\alpha)$. Conditions (2) often represent compliance with standard regulatory requirements to the system. As a rule, vectors of design variables $\alpha = (\alpha_1, \alpha_2, ..., \alpha)_r$ are calculated using uniformly distributed sequences. In the present case, a random number generator was applied because of high dimensionality of the design variable space.

There also exist particular *performance criteria*. It is desired, all other things being equal, these criteria, denoted by $\Phi_\nu(\alpha)$, $\nu = 1,..,k$, take the extreme values. For simplicity, we assume that $\Phi_\nu(\alpha)$ are to be minimized.

The constraints (1) single out a parallelepiped Π in the r-dimensional design variable space (space of design variables).

In order to avoid situations, in which the expert regards the values of some criteria as unacceptable, we introduce the *criteria constraints*

$$\Phi_\nu(\alpha) \le \Phi_\nu^{**}, \nu = 1,..,k, \quad (3)$$

where Φ_ν^{**} is the worst value of the criterion $\Phi_\nu(\alpha)$ to which the expert may agree. The choice of $\Phi_\nu(\alpha)$ is discussed in what follows.

The criteria constraints differ from the functional constraints in that the former are determined when solving a problem and, as a rule, are repeatedly revised. Hence, unlike C_l^* and C_l^{**}, reasonable values of Φ_ν^{**} cannot be chosen before solving the problem.

Constraints (1)-(3) define the feasible solution set D.

If the functions $f_l(\alpha)$ and $\Phi_\nu(\alpha)$ are continuous in Π, then the sets G and D are closed.

Now let us formulate one of the basic problems of multicriteria optimization. It is necessary to find such a set $P \subset D$ for which

$$\Phi(P) = \min_{\alpha \in D} \Phi(\alpha) \quad (4)$$

where $\Phi(\alpha) = (\Phi_1(\alpha), \Phi_2(\alpha), ..., \Phi_k(\alpha))$ is the criterion vector and P is the Pareto optimal set.

In other words, a point $\alpha^0 \in D$, is called a Pareto optimal point if there exists no point $\alpha \in D$ such that $\Phi_\nu(\alpha) \le \Phi_\nu(\alpha^0)$ for all $\nu = 1, \ldots, k$ and $\Phi_{\nu 0}(\alpha) \le \Phi_{0\nu}(\alpha^0)$ for at least one $\nu \in \{1, \ldots, k\}$. A set $P \subset D$ is called the Pareto optimal set iff it consists of Pareto optimal points.

The Pareto optimal set plays an important role in vector optimization problems, because it can be analyzed relatively easier than the feasible solution set and because, by definition, the optimal vector always belongs to the Pareto optimal set, irrespective of the system of preferences used by the expert for comparing vectors belonging to the feasible solution set.

Very often, the experts do not encounter serious difficulties in analyzing the feasible solution set and the optimal set and in choosing the most preferred solution. They have a sufficiently well-defined system of preferences. Moreover, the aforementioned sets usually contain a small number of elements [17,23].

3.2. The PSI Method

To formulate and solve engineering optimization problems, the Parameter Space Investigation (PSI) method has been developed. According to this method, in the process of dialogues with a computer, the expert determines the criteria constraints and performs multicriteria analysis. The PSI method gives the expert valuable information on the advisability of revising various criteria, functional, and design variable constraints with the aim of improving the basic criteria. The expert sees what price one pays for making concessions in various criteria, *i.e.*, what one loses and what one gains. In other words, the expert corrects initial problem statement while solving it, analyses the feasible solution set, and then makes a decision. A systematic and comprehensive description of the method can be found in [17,21,23].

After analyzing *P* (Pareto optimal set), the expert finds the most preferred solution $\Phi(\alpha^0)$. Typically, for the problems under consideration, experts do not have serious difficulties in analyzing the Pareto optimal set and in choosing the most preferred solution. Thus, the PSI method has proved to be a very convenient and effective tool for the expert, primarily because this method can be directly used for the statement and solution of the problem in an *interactive mode***.** The PSI method is implemented in the Multicriteria Optimization and Vector Identification (MOVI) software system [17].

It is also worth mentioning that while there are many optimization methods, the PSI method more fully addresses characteristics of real-world engineering optimization problems (e.g., multiple criteria, difficulties in determining constraints on design variables, functional dependences and criteria) and allows the expert to simultaneously formulate and solve them in an interactive mode.

4. Application of the PSI Method to Improving the Network

The PSI method has been applied to the mathematical model of the network described above. Recall that in the above two examples, we have 50 cells with 100 variables and the number of criteria is seven. As test examples, we solve Task 1 and Task 2, that differ in maximum allowed transmit power—40 W and 50 W, respectively.

4.1. Test Tables for Task 1 (40 W)

As follows from the PSI method, the criteria constraints should be determined first. We have constructed the test table after 10,000 tests, see **Table 2**. The list of criteria, the best and worst values of criteria are shown in the

Table 2. Fragment of the test tables (Criteria that maximized and minimized are denoted with ↑ and ↓, respectively).

CDBR ↓		RSRPcovA ↑		RSSINRcovA ↑		RSRPcovT ↑		RSSINRcovT ↑		MeanSINR ↑		MeanLoad ↓	
Best	0.01074	Best	0.97573	Best	0.60156	Best	0.97532	Best	0.62937	Best	0.94030	Best	0.32277
Worst	0.17222	Worst	0.68947	Worst	0.39751	Worst	0.71008	Worst	0.40852	Worst	0.52068	Worst	0.54562
Vector#	Value	Vector#	Value	Vector#	Value	Vector#	Value	Vector#	Value	Vector#	Value	Vector#	Value
...		...		...		...		...		...		...	
8839	0.06997	2814	0.89004	166	0.53004	858	0.91019	8986	0.55007	580	0.75007	6293	0.38996
5500	0.06998	9482	0.89004	4437	0.53000	993	0.91015	7222	0.55007	1397	0.75007	3835	0.38996
4806	0.06999	5614	0.89004	8962	0.53000	5693	0.91011	7415	0.55005	5386	0.75007	836	0.38998
7415	0.07000	1259	0.89004	6618	0.53000	6551	0.91010	5896	0.55004	3324	0.75006	129	0.38998
8127	0.07000	9952	0.89000	5982	0.53000	4405	0.91010	2046	0.55002	5346	0.75001	920	0.38999
8556	**0.07000**	**4299**	**0.89000**	**3288**	**0.53000**	**4575**	**0.91003**	**6578**	**0.55001**	**2998**	**0.75000**	**9311**	**0.38999**
85	0.07000	5142	0.88996	1710	0.52996	7821	0.90999	7436	0.54999	8530	0.74998	5434	0.39000
3377	0.07001	1751	0.88996	347	0.52991	7426	0.90996	6051	0.54999	8804	0.74997	5063	0.39001
5038	0.07001	5985	0.88996	1260	0.52991	2453	0.90994	2943	0.54998	4952	0.74996	7360	0.39002
9599	0.07002	1077	0.88996	7500	0.52991	7408	0.90989	1417	0.54998	6012	0.74994	7427	0.39003
2643	0.07003	4531	0.88991	6774	0.52991	7284	0.90987	8178	0.54997	6711	0.74994	5420	0.39004
...		...		...		...		...		...		...	

first, second and third rows, respectively. As a result of the dialogues of computer with the expert, the criteria constraints (0.07000; 0.89000; 0.53000; 0.91003; 0.55001; 0.75000; 0.38999) have been determined, see **Table 2**. We have obtained 256 feasible solutions, including 50 Pareto optimal solutions. Since these constraints met the expert's requirements, they have been accounted for in all further studies. These constraints were identical in Task 2 (50 W), where we have conducted 10,000 tests and obtained 2867 feasible solutions, including 95 Pareto optimal solutions.

4.2. Criteria Histograms for Task 1

Criteria histograms are constructed on the basis of test tables and allow us to make a decision based on the obtained values of the criteria vectors and their significance [17,23]. In particular, the histograms allow us to correct the initial problem statement and reveal criteria relations. Histograms show Pareto optimal vectors for all criteria (e.g., see **Figure 2**). For each criterion, a horizontal line is assigned, along which vertical bars are plotted. Each bar corresponds to a vector from the Pareto optimal set. The location of each bar is defined by a corresponding criterion value for this vector. Criterion name as well as the worst and best criterion values are displayed to the left and to the right of the corresponding horizontal band, respectively.

Figure 2 shows locations of all 50 Pareto optimal vectors within obtained constraints on their values. Each of 50 solutions is uniquely colored. As can be seen, improving some criteria leads to deterioration of the others. For example, Pareto optimal solution #2544 is the best according to the fourth criterion (RSRP-covT, $\Phi_4^{2544} = 0.9708$) and one of the best according to the second criterion (RSRP-covA, $\Phi_2^{2544} = 0.9535$) however it is the worst according to the seventh criterion (M-load, $\Phi_7^{2544} = 0.3868$) and one of the worst according to the third (RSSINR-covA, $\Phi_3^{2544} = 0.5355$), fifth (RSSINR-covT, $\Phi_5^{2544} = 0.5533$), and sixth (Mean-SINR, $\Phi_6^{2544} = 0.7692$) criteria, (see **Figure 2**). These histograms demonstrate complex relationships that exist between the criteria.

4.3. Search for Optimal Solutions by Optimization of the Criteria Aggregate

In our case, the biggest challenges for an expert included the choice of a preferred solution from the Pareto optimal set: there were 50 and 95 Pareto optimal solutions in Task 1 and Task 2, respectively. As mentioned above, an expert defined an objective function as an aggregate of criteria

$$\Phi_{\text{aggr}} = \sum_{i=1}^{7} \beta_i \Phi_i,$$

where $\beta_1 = 0.2$, $\beta_2 = -0.2$, $\beta_3 = -0.1$, $\beta_4 = -0.2$, $\beta_5 = -0.1$, $\beta_6 = -0.1$, and $\beta_7 = 0.1$. The search for optimal solution was performed on the feasible set which is defined by constraints on parameters and criteria. For Task 1, the minimal value of the criteria aggregate (see [24]) was –0.6226, with the vector of criteria (0.0072; 0.9804; 0.6803; 0.9827; 0.7207; 1.1813; 0.2676). For Task 2, the minimal value of the criteria aggregate was –0.6262 with the vector of criteria (0.0108; 0.9748; 0.6825; 0.9819; 0.7275; 1.2209; 0.2612).

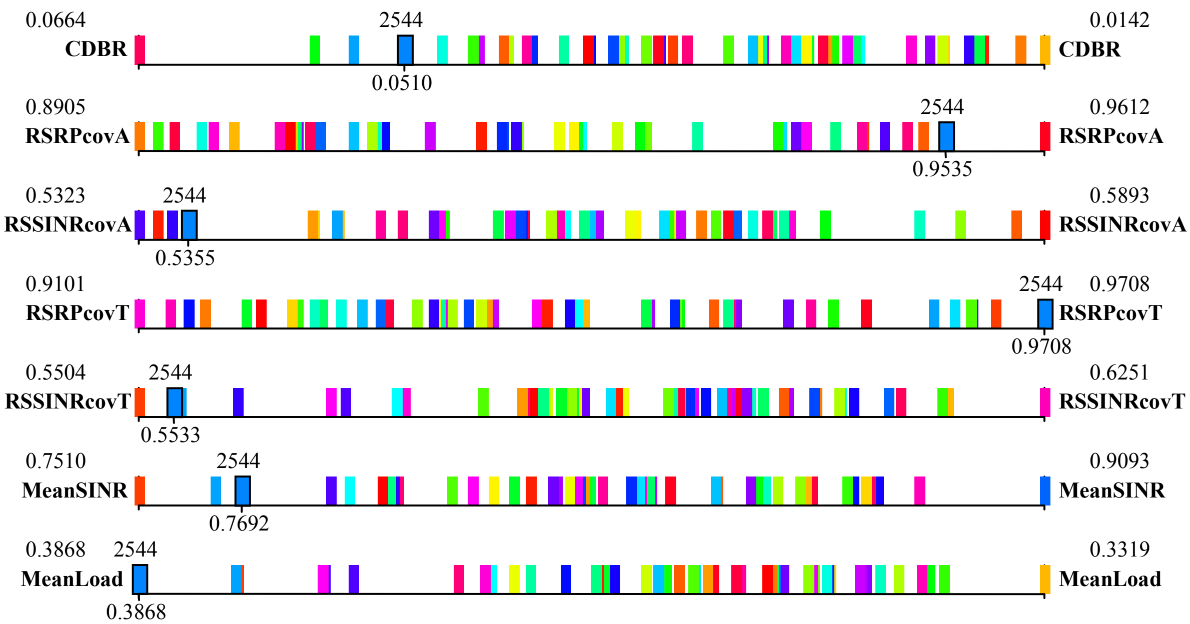

Figure 2. Criteria histograms. Location of the vector #2544. See text for details.

5. Conclusions

The conclusions of this work are three-fold. First, defining the feasible solution set is directly related to the correct statement of the optimization problem for the cellular network. We developed methodology for stating and solving the problems of multicriteria optimization for cellular network and showed how to state and solve the multicriteria problem of construction of the feasible and Pareto optimal solution sets.

Second, as it is known, the most preferable solution is determined on the Pareto set. In order to find it, one has to determine feasible solution set. Determination of the latter, correct definition of criteria constraints, turned out to be a difficult problem. These constraints were determined as a result of dialogues between an expert and compute while analyzing the test tables. Analysis of the Pareto optimal set is also challenging for the expert because of a large number of Pareto solutions. To overcome this, we have used a convolution of criteria. It is worthwhile to note that only after the feasible solution set has been determined, the criteria convolution can be successfully applied to find the optimal solution.

Third, to tune the network parameters online, *i.e.* to account for real distribution of the traffic, the operator may reduce the time required for the analysis. This process can be automated.

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