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Multicriteria analysis of real-life engineering optimization problems: statement and solution

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Abstract

The majority of engineering problems are essentially multicriteria. These criteria are usually contradictory. That is why specialists experience significant difficulties in correctly stating engineering optimization problems, so designers often end up solving ill-posed problems. In general, it is impossible to reduce multicriteria problems to single-criterion ones.

For the correct statement and solution of engineering optimization problems, a method called *Parameter Space Investigation* (PSI method) has been created and widely integrated into various fields of industry, science, and technology (e.g., design of the space shuttle, nuclear reactor, missile, automobile, ship, and metal-tool). In summary, the PSI method generates many feasible designs from which the so-called Pareto optimal ones (i.e. solutions which cannot be improved) are extracted. The PSI method can also be used to efficiently optimize models in a parallel mode, which is of great importance while solving high-dimensional multiparameter and multicriteria problems.

The PSI method is implemented in the software package *Multicriteria Optimization and Vector Identification* (MOVI), a comprehensive system for multicriteria engineering optimization (design, identification, and control). This system allows optimization of many problems that until recently appeared intractable.

Keywords: Multicriteria analysis; PSI method; Feasible solution set; Pareto optimal set; MOVI software system

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1. Introduction

We discuss here the methodology of mathematical formulation and solution that can be applied to the basic real-life optimization problems: design, identification, design of controlled systems, operational development of prototypes, finite-element models, optimization of large-scale systems, etc.

Many optimization methods are used for the search for optimal solutions. For example, common techniques are nonlinear programming [1,3] and genetic algorithms [2,4]. When such methods are applied, it is tacitly assumed that the user can correctly state the problem and thus determine the feasible solution set. Unfortunately, this is not the case in reality. *Existing optimization methods are not designed to formulate the problem*. Therefore, in the majority of cases, the user ends up solving ill-posed problems.

The determination of the feasible solution set is a fundamental element in engineering optimization. The *Parameter Space Investigation* (PSI) method is designed to determine the feasible solution set and has been successfully applied to various fields of human activity [5,6].

2. Formulation and solution of multicriteria optimization problems

For the detailed review of the PSI method, refer to [5,6]. Below, we discuss formulation of the multicriteria optimization problem and summarize the main ideas of PSI method.

Consider an object described by a system of equations (e.g., differential, algebraic, etc.) with some performance criteria. We assume that performance criteria of the object depend on *r design variables* $\alpha_1, \ldots, \alpha_r$ representing a point $\alpha = (\alpha_1, \ldots \alpha_r)$ in the r-dimensional space. In order to formulate a multicriteria optimization problem correctly, one has to impose constraints on design variables, functions and criteria.

The *design variable constraints* can be written as $\alpha_j^* \leqslant \alpha_j \leqslant \alpha_j^{**}$, $j = 1, \ldots, r$. In case of mechanical systems, design variables α_j can represent stiffness coefficients, moments of inertia, masses, damping factors, geometric dimensions, etc. The constraints α_j^* and α_j^{**} define a parallelepiped Π in the r-dimensional design variable space.

The *functional constraints* can be written in the form $c_l^* \leqslant f_l(\alpha) \leqslant c_l^{**}$, $l = 1, \ldots, t$, where $f_l(\alpha)$ is a functional dependency, c_l^* and c_l^{**} are some constants, and t is the number of functions.

We seek to minimize[1] local-performance criteria $\Phi_\nu(\alpha)$, $\nu = 1, \ldots, k$. In order to avoid situations when the values of certain criteria are inappropriate from the viewpoint of an expert, *criteria constraints* must be introduced: $\Phi_\nu(\alpha) \leqslant \Phi_\nu^{**}$, where Φ_ν^{**} is the marginal value of a criterion $\Phi_\nu(\alpha)$ acceptable by an expert.

The functional dependencies $f_l(\alpha)$ and the performance criteria $\Phi_\nu(\alpha)$ may be either implicit or explicit functions of α. In other words, for some systems, we may know the exact mathematical dependencies of f_l and Φ_ν on α, whereas sometimes f_l and Φ_ν act as "black-boxes".

[1] In the description of our methodology, we consider only the case of minimization; maximization works similarly.

We would like to emphasize that the criteria constraints Φ_v^{**} are usually determined during the solution of the problem and, as a rule, can be repeatedly revised. The functional constraints c_l^* and c_l^{**} are often rigidly specified and are constant (e.g., standards or specifications can be functional constraints). Some functional constraints, however, can be "soft", in a sense that they can change. We refer to the latter functional constraints as *pseudocriteria*. Pseudocriteria are not considered while constructing Pareto optimal solutions.

All the constraints on design variables, functions, and criteria define the feasible solution set D, $D \subseteq \Pi$.

In the basic problems of multicriteria optimization, it is necessary to find a set $P \subset D$ for which $\Phi(P) = \min_{\alpha \in D} \Phi(\alpha)$, where $\Phi(\alpha) = (\Phi_1(\alpha), \ldots, \Phi_k(\alpha))$ is the criterion vector and P is the *Pareto optimal set*. We mean that $\Phi(\alpha) < \Phi(\beta)$ if for all $v = 1, \ldots, k$, $\Phi_v(\alpha) \leqslant \Phi_v(\beta)$ and for at least one $v_0 \in \{1, \ldots, k\}$, $\Phi_{v_0}(\alpha) < \Phi_{v_0}(\beta)$. When solving the problem, one has to determine design variable vector $\alpha^0 \in P$, which is the *most preferable* among vectors belonging to the set P.

The Pareto optimal set plays an important role in multicriteria engineering problems, since this set can be analyzed more easily than the feasible solution set and it contains only optimal solutions. The importance of Pareto optimal set is determined to a great extent by the following theorem (proved in Ref. [5]):

Theorem. *If feasible solution set D is closed and criteria* $\Phi_v(\alpha)$ *are continuous*, *then the Pareto optimal set is nonempty*.

While solving problems with contradictory criteria (as is often the case for engineering optimization problems), an expert cannot a priori formulate criteria constraints Φ_v^{**} correctly. The same is often true for pseudocriteria c_l^* and c_l^{**}. Furthermore, the determination of design variable constraints (i.e., α_j^* and α_j^{**}), which are not specified rigidly, is a challenging task. The traditional optimization methods are not designed to solve the above problems and thus fail to formulate the optimization problem correctly. In this sense, the key task of proposed methodology is to formulate the optimization problem based on determination of the feasible set D.

2.1. Parameter Space Investigation (PSI) method

The PSI method is based on the search of the parallelepiped Π with points of uniformly distributed sequences (e.g., LP_τ sequences), pseudo-random uniform numbers, etc. (see [5,6] for details). If the functional constraints c_l^* and c_l^{**}, $l = 1, \ldots, t$ are satisfied for a vector of design variables α^i, we proceed to the computation of performance criteria $\Phi_v(\alpha^i)$, $v = 1, \ldots, k$. The process of generating design variable vectors with the following computation of functional constraints is repeated N times. We refer to N as the *number of trials*.

The parameter (or design variable) space is investigated in three stages. In the *first stage*, we construct *test tables* by performing N trials and arranging the values of $\Phi_v(\alpha^1), \ldots, \Phi_v(\alpha^N)$ in ascending order (assuming that all the criteria must be minimized). In the *second stage*, an expert chooses preliminarily criteria constraints Φ_v^{**}, $v = 1, \ldots, k$, which are the maximum values of the criteria $\Phi_v(\alpha)$, for which an acceptable level of the object's performance is guaranteed. In the *third stage*, the problem's solvability is verified. In other words, the vectors α^i satisfying simultaneously all the inequalities $\Phi_v(\alpha^i) \leqslant \Phi_v^{**}$, $v = 1, \ldots, k$, are determined. If the set of these vectors is nonempty, then the problem of construction of the feasible set is solvable. Otherwise, one has to either modify the values of Φ_v^{**} or return to the first stage and increase the number of trials (in order to repeat the second stage given a larger test table). The procedure is repeated until D is nonempty and the maximum values of Φ_v^{**} are determined. After that, the Pareto set is constructed and analyzed in accordance with [5,6].

The above-described multicriteria problem can be viewed as the basis for analyzing other engineering problems: design, identification, design of controlled systems, operational development of prototypes, finite-element models, optimization of large-scale systems, etc.

2.2. Implementation of the PSI method

The PSI method is implemented in MOVI (*Multicriteria Optimization and Vector Identification*), a comprehensive software system for multicriteria engineering optimization. The software package MOVI is designed to apply the PSI method to a wide range of engineering problems.

For the PSI method and MOVI to be successful when applied to real-life problems, the software (and methodology) should scale to large systems with as many as thousands of design variables and dozens of criteria. The software package MOVI allows to tackle these problems in a *parallel mode* so that the desired number of trials N is distributed between k computers (nodes). Thus, each node finds a feasible solution set for its own subproblem (by conducting $\approx N/k$ trials). Next, all feasible solution sets are combined and a single Pareto optimal solution set is constructed.

In summary, the current version (1.3) of MOVI allows up to 51 design variables in the problems with $LP\tau$ sequences and thousands of design variables with pseudo-random number generators (RNG). The number of criteria to be optimized is limited only by the computer's processing power. The number of criteria reached many dozens when we solved real-life problems (e.g., a 65-criteria identification problem of operational development of a vehicle was solved by application of PSI method and is described in Ref. [5]).

The software package MOVI is also aimed at simplifying the interpretation of the results of parameter space investigation and contains a number of visualization elements, such as graphs, tables, and histograms. These elements allow the designer to (1) estimate dependencies of criteria on the design variables and dependencies between criteria, (2) determine significant criteria, (3) compare the values of criteria for a baseline design (prototype) with the results obtained by the PSI method, and (4) judge whether it is advisable or not to correct the initial statement of the problem and/or perform further trials to improve the results of optimization. Below we describe four central elements of visualization.

2.2.1. Histograms of feasible solutions

Visualization of the distribution of feasible solutions over the design variable intervals $[\alpha_j^*, \alpha_j^{**}]$, $j = 1, \ldots, r$ is of great importance. In particular, the histograms show the role of the functional and criteria constraints in the design variable space and allow designer to correct the initial design variable constraints accordingly.

2.2.2. Graphs criterion vs. design variable I

After the analysis of the test table, the preference was given to design variable vector α^i. We fix all components of this vector except for one, α^i_j, and find out how the criteria $\Phi_1, \ldots, \Phi_k$ change as the component α^i_j varies in the initial interval $[\alpha^*_j, \alpha^{**}_j]$. This analysis is most frequently used for investigation of Pareto optimal solutions.

2.2.3. Graphs criterion vs. design variable II

Assume that after N trials, $N1$ design variable vectors have entered the test table. We consider projections of the points $(\Phi_v(\alpha^i), \alpha_1, \ldots, \alpha_r)$, $v = 1, \ldots, k$, $i = 1, \ldots, N1$ onto the plane $\Phi_v \alpha_j$, $j = 1, \ldots, r$. These projections provide an expert with information about dependencies of a criterion on design variables.

2.2.4. Graphs criterion vs. criterion

Again, assume that after N trials, $N1$ design variable vectors have entered the test table. We consider projections of the points $\Phi_v(\alpha^i)$, $v = 1, \ldots, k$, $i = 1, \ldots, N1$ onto the plane $\Phi_m \Phi_n$, where m and $n \in \{1, \ldots, k\}$. These projections provide an expert with information about dependencies between criteria.

3. Experiments: construction of the feasible sets

3.1. Two-mass dynamical system

In this example, we determine the feasible solution set of the two-mass dynamical system shown in Fig. 1. The system consists of two bodies with masses M_1 and M_2. The mass M_1 is attached to a fixed base by a spring with stiffness coefficient K_1. A spring-and-dashpot element with stiffness coefficient K_2 and damping coefficient C is located between masses M_1 and M_2. The harmonic force $P \cdot \cos(\omega t)$ acts upon mass M_1. The amplitude and frequency of the exciting force are identified as $P = 2000\,\mathrm{N}$ and $\omega = 30\,\mathrm{s}^{-1}$. The motion of this system is governed by the equations:

$$\begin{aligned} &M_1 X_1'' + C(X_1' - X_2') + K_1 X_1 + K_2(X_1 - X_2) = P \cdot \cos(\omega t), \\ &M_2 X_2'' + C(X_2' - X_1') + K_2(X_2 - X_1) = 0. \end{aligned} \tag{1}$$

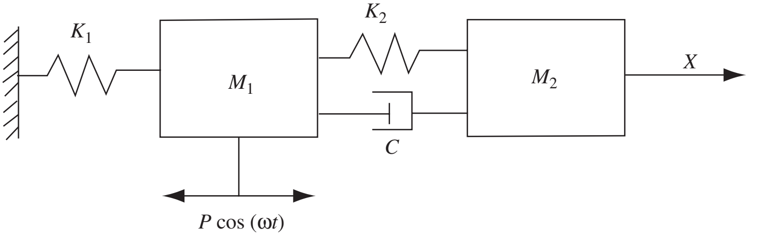

Fig. 1. Two-mass dynamical system.

We treat the parameters K_1, K_2, M_1, M_2, and C as the design variables to be determined, i.e., $a_1 = K_1$, $a_2 = K_2$, $a_3 = M_1$, $a_4 = M_2$, and $a_5 = C$. The design variable constraints are prescribed as the parallelepiped Π defined by the inequalities:

$$\begin{aligned}
&1.1 \times 10^6 \leqslant \alpha_1 \leqslant 2.0 \times 10^6 \ \ (\mathrm{N/m}),\\
&4.0 \times 10^4 \leqslant \alpha_2 \leqslant 5.0 \times 10^4 \ \ (\mathrm{N/m}),\\
&950 \leqslant \alpha_3 \leqslant 1050\,(\mathrm{kg}),\\
&30 \leqslant \alpha_4 \leqslant 70\,(\mathrm{kg}),\\
&80 \leqslant \alpha_5 \leqslant 120\,(\mathrm{N\,s/m}).
\end{aligned} \tag{2}$$

There are three functional constraints (on the total mass and the partial frequencies):

$$\begin{aligned}
&f_1(\alpha) = \alpha_3 + \alpha_4 \leqslant 1100\,(\mathrm{kg}),\\
&33 \leqslant f_2(\alpha) = p_1 = \sqrt{\alpha_1/\alpha_3} \leqslant 42\,(\mathrm{s}^{-1}),\\
&27 \leqslant f_3(\alpha) = p_2 = \sqrt{\alpha_2/\alpha_4} \leqslant 32\,(\mathrm{s}^{-1}).
\end{aligned} \tag{3}$$

The upper limits imposed on the functions $f_2(\alpha)$ and $f_3(\alpha)$ are not rigid. For this reason, the functional relations $f_2(\alpha)$ and $f_3(\alpha)$ are interpreted as pseudocriteria Φ_1 and Φ_2, respectively. Thus, we have three functional constraints:

$$\begin{aligned}
&f_1(\alpha) = \alpha_3 + \alpha_4 \leqslant 1100,\\
&33 \leqslant f_2(\alpha),\\
&27 \leqslant f_3(\alpha).
\end{aligned} \tag{4}$$

We want to optimize the system with respect to the following four performance criteria:

- $\Phi_3 = X_{1\partial}$ (mm)—vibration amplitude of the first mass;
- $\Phi_4 = M_1 + M_2$ (kg)—metal consumption of the system;
- $\Phi_5 = X_{1\partial}/X_{1st}$—dimensionless dynamical characteristic of the system;
- $\Phi_6 = \omega/p_1$—dimensionless dynamical characteristic of the system,

where X_{1st} is the static displacement of mass M_1 under the action of the force P. Thus, we have a vector of criteria $\Phi = (\Phi_1, \Phi_2, \Phi_3, \Phi_4, \Phi_5, \Phi_6)$, which will be used for construction of the test tables.

3.1.1. Experiment 1: Is the statement of the problem correct?

We performed 1024 trials using LP_τ sequences and constructed the test table. 789 solutions were included in the test table since they satisfied functional constraints. While analyzing the test table, the following criteria constraints were formulated:

$$\begin{aligned}
&\Phi_1^{**} = 35.2008,\\
&\Phi_2^{**} = 36.9807,\\
&\Phi_3^{**} = 8.4166,\\
&\Phi_4^{**} = 1019.1211,\\
&\Phi_5^{**} = 18.795,\\
&\Phi_6^{**} = 0.9087.
\end{aligned} \tag{5}$$

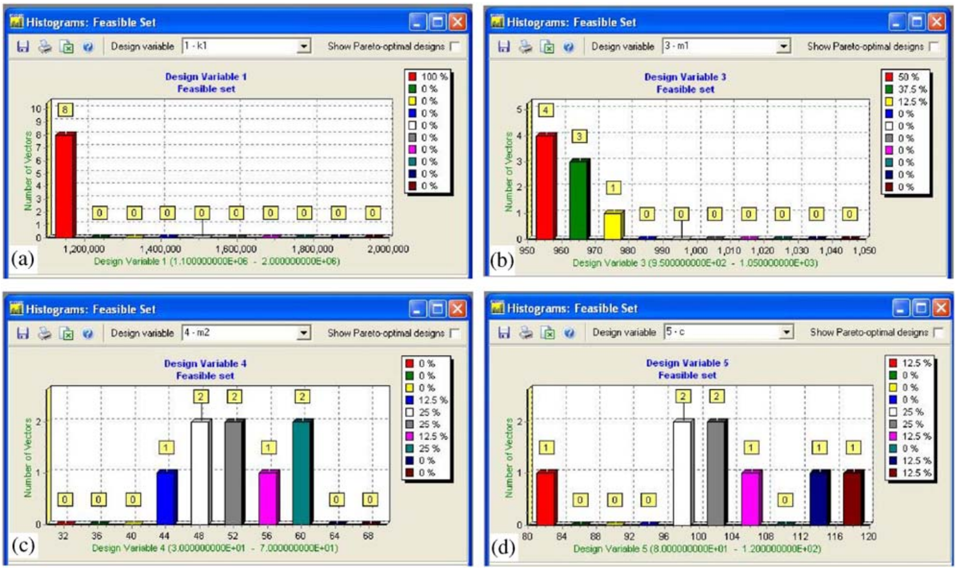

Fig. 2. Histograms of the distribution of feasible solutions.

Table 1
Refining initial design variables constraints

Initial intervals of variation of design variables (Experiment 1)	Subintervals where the feasible solutions belong (Experiment 1)	New intervals of variation of design variables (Experiment 2)
$1.1 \times 10^6 \leqslant \alpha_1 \leqslant 2.0 \times 10^6$	$1.1 \times 10^6 \leqslant \alpha_1 \leqslant 1.17 \times 10^6$	$9 \times 10^5 \leqslant \alpha_1 \leqslant 1.2 \times 10^6$
$950 \leqslant \alpha_3 \leqslant 1050$	$950 \leqslant \alpha_3 \leqslant 975$	$850 \leqslant \alpha_3 \leqslant 980$
$30 \leqslant \alpha_4 \leqslant 70$	$42 \leqslant \alpha_4 \leqslant 60.35$	$40 \leqslant \alpha_4 \leqslant 64$

Only eight solutions were found to be feasible (i.e. satisfied constraints (5)). Four out of these feasible solutions are Pareto optimal solutions corresponding to trials #520, #336, #672 and #288.

The analysis of histograms shows the effect of functional and criteria constraints (see Figs. 2(a)–(d)). In particular, all feasible solutions for design variables α_1 and α_3 are located in the left ends of the intervals (Figs. 2(a),(b)). The feasible solutions for the design variable α_4 are located in the middle of the interval (Fig. 2(c)). On the other hand, the feasible solutions for α_2 and α_5 are more or less uniformly distributed along the interval (see Fig. 2(d) for the histogram for α_5). These histograms were produced in MOVI using the option *Histograms of feasible solutions*.

The analysis of histograms for design variables α_1, α_3, and α_4 is summarized in Table 1. The first column of Table 1 lists the initial intervals of variation of α_1, α_3, and α_4. The second column contains the corresponding subintervals, where the feasible solutions belong.

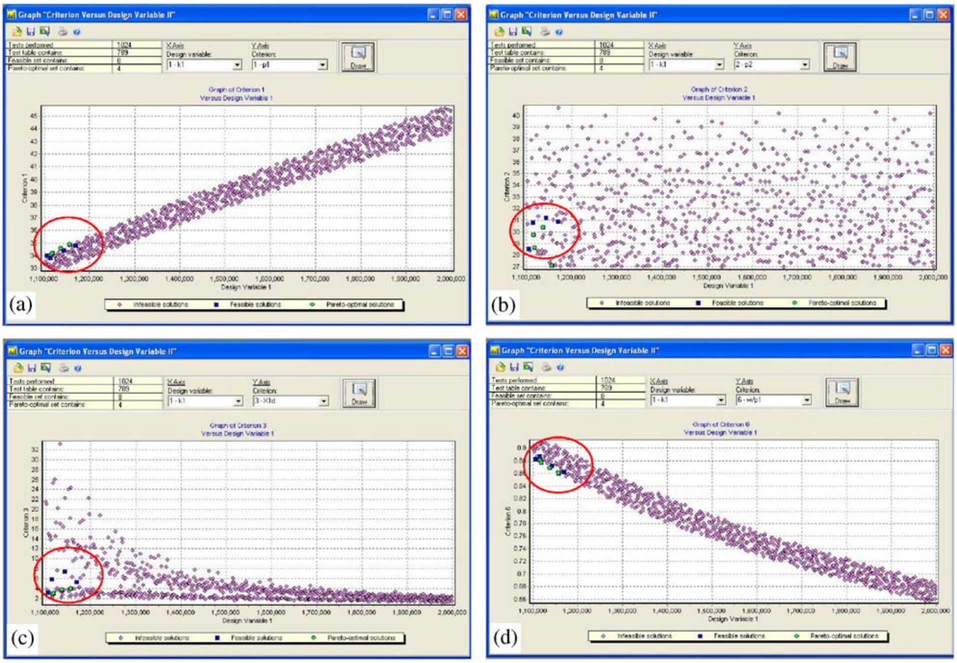

Fig. 3. Dependencies of criteria on the design variable.

In order to improve obtained feasible solutions, an expert decided to redo the investigation with the modified initial intervals of variation of design variables α_1, α_3, and α_4 (as shown in the last column in Table 1) and initial intervals (i.e., as in (2)) for α_2 and α_5. This defines a new parallelepiped Π_1 that was used for Experiment 2 (see Section 3.1.2).

Another important aspect of the analysis is to study the influence of design variables on criteria. For example, Figs. 3(a)–(d) show dependencies of criteria Φ_1, Φ_2, Φ_3, and Φ_6 on design variable α_1, respectively. We can conclude from the Figs. 3(a) and (d) that criteria Φ_1 and Φ_6 are controversial with respect to α_1. The criterion Φ_3 is also dependent on α_1 (Fig. 3(c)), while the dependency of Φ_2 on α_1 is not obvious (Fig. 3(b)). These figures were produced in MOVI using the option *Graphs Criterion vs. Design Variable II*. The feasible and Pareto optimal solutions are circled in the figures.

In order to make decisions about the most preferable solution in Pareto set, it is necessary to analyze dependencies between criteria (Fig. 4). For example, Figs. 4(a)–(c) show dependencies between Φ_1 and Φ_6, Φ_3, and Φ_2, respectively. Fig. 4(d) shows dependency between Φ_2 and Φ_3. These figures were produced in MOVI using the option *Graphs Criterion vs. Criterion*. The feasible and Pareto optimal solutions are circled in the figures.

After the analysis of Pareto optimal solutions, an expert decided to use design variable vector #288. Fig. 5 shows the dependencies of criteria on design variables for vector #288 (when one design variable is changing all the remaining ones are fixed). We can see that criteria Φ_1 and Φ_3 are antagonistic with respect to α_1 (see Figs. 5(a), (b), respectively).

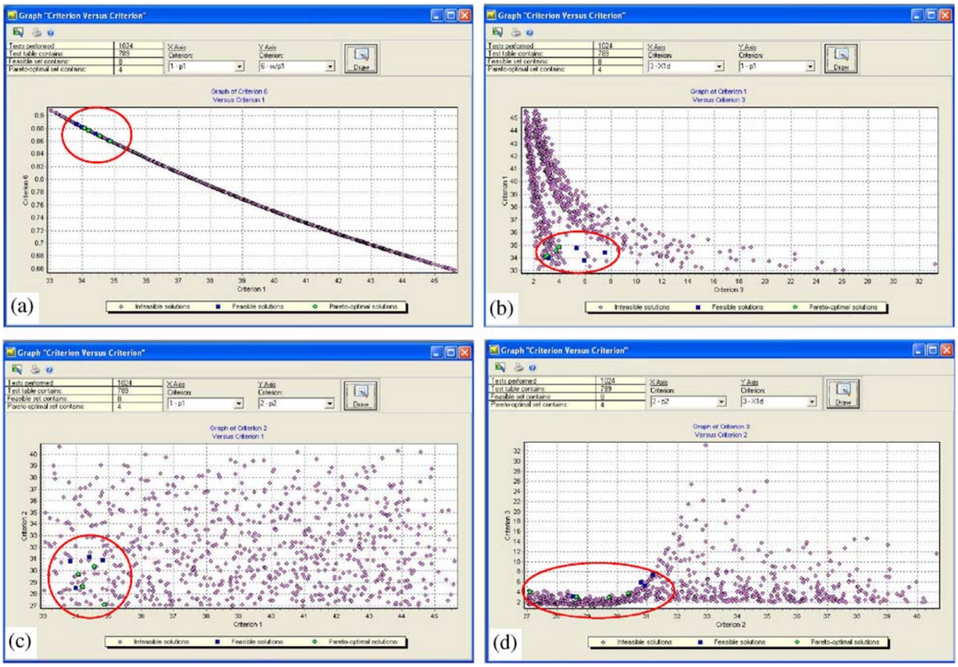

Fig. 4. Dependencies between criteria.

Similarly, criteria Φ_1 and Φ_6 are antagonistic with respect to α_3 (see Figs. 5(c), (d), respectively). These figures were produced in MOVI using the option *Graphs Criterion vs. Design Variable I*. The feasible and Pareto optimal solutions are circled in the figures.

3.1.2. Experiment 2: Improving feasible solution set by changing initial intervals of variation of design variables

In this experiment, we are seeking to improve the feasible solution set obtained in Experiment 1 by using a new parallelepiped Π_1. The functional and criteria constraints were the same in both experiments. After 1024 tests using LP_τ sequences, the number of feasible solutions is 258 (compared to 8 in Experiment 1), and the number of Pareto optimal solutions is 25 (compared to 4 in the previous experiment). Next, we combined feasible solution sets from both experiments (this option is also available in MOVI) and determined Pareto optimal solutions on the combined feasible solution sets. The combined Pareto optimal set contained only 25 solutions, and all of them were obtained in Experiment 2. Thus, all solutions from Experiment 1 were improved as it follows from the definition of Pareto optimal solution set.

3.1.3. Experiment 3: Improving feasible solution set by correcting functional constraints

After analysis of the *table of functional failures* (i.e., the list of designs which do not satisfy functional constraints) obtained from Experiment 2, an expert agreed for a concession by

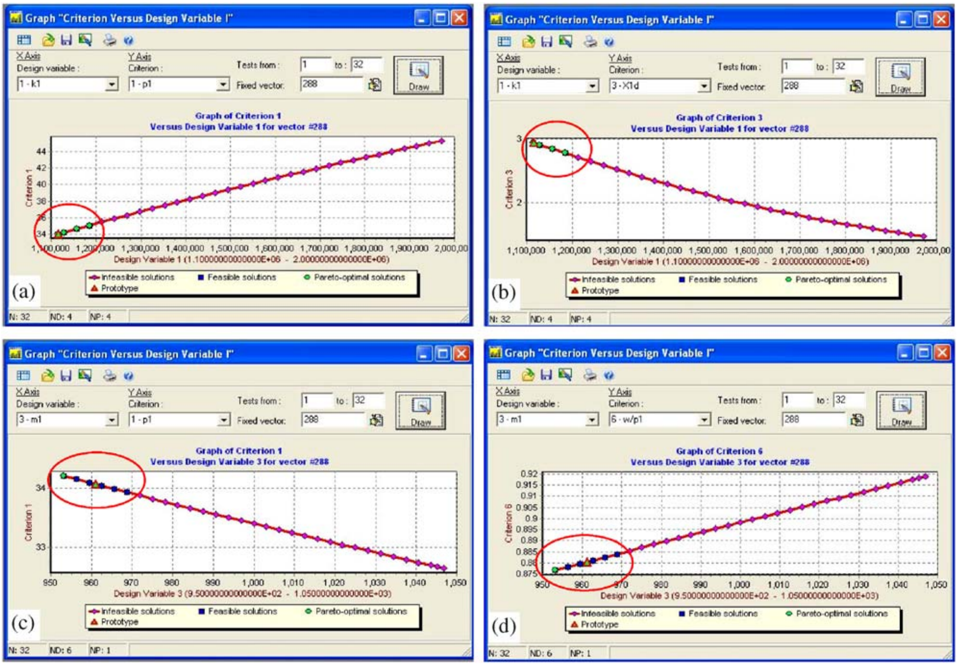

Fig. 5. Dependencies of criteria on design variables for Pareto optimal solution /#288.

changing functional constraints for f_2 and f_3 from $33 \leqslant f_2(\alpha)$, $27 \leqslant f_3(\alpha)$, to $32.5 \leqslant f_2(\alpha)$ and $26.5 \leqslant f_3(\alpha)$. As a result, the number of feasible and Pareto optimal solutions increased to 282 and 26, respectively.

3.1.4. *Commentary on expert's behavior*

Most often, an expert preliminarily agrees with modification of the initial statement of the problem (e.g., by changing design variable, functional and criteria constraints). In other words, an expert can be interested in *how to modify constraints in order to improve the values of criteria of interest*. The final decision is made only after the analysis of obtained results (i.e. by judging whether the obtained improvement of the main performance criteria is worthy of concessions).

3.2. *Multicriteria optimization of a large-scale system in parallel mode*

Many engineering problems contain as many as hundreds and thousands of design variables. Below, we show how the proposed methodology tackles these problems in a parallel mode by considering a hypothetical example of optimization of a large-scale system. Consider a large-scale system with 1000 design variables. The design variable vector is given by $\alpha = (\alpha_1, \ldots, \alpha_{1000})$, $1 \leqslant \alpha_i \leqslant 2$, $i = 1, \ldots, 1000$. We are seeking to minimize simultaneously the following performance criteria $\Phi_v(\alpha)$, $v = 1, \ldots, 4$:

$$
\begin{aligned}
\Phi_1 &= \sum_{1}^{1000} \alpha_i, \\
\Phi_2 &= \sum_{300}^{1000} \alpha_i^2 - \sum_{1}^{299} \alpha_i^2, \\
\Phi_3 &= \left(1400 \Big/ \sum_{300}^{1000} \alpha_i^2\right) - \cos\left(\sum_{1}^{299} \alpha_i\right), \\
\Phi_4 &= \sum_{1}^{700} \frac{\alpha_i}{i} - \left(\sin\left(\sum_{701}^{1000} \alpha_i^2\right)\right)^5 .
\end{aligned}
\tag{6}
$$

While analyzing the test tables, we formulated the following criteria constraints:

- $\Phi_1 < 1502.2254$,
- $\Phi_2 < 930.4528$,
- $\Phi_3 < 0.1624$,
- $\Phi_4 < 10.3851$.

We investigated the parameter and criteria space on four nodes simultaneously. Each node conducted 50,000 trials using RNG. Totally, the four nodes conducted 200,000 trials, which resulted in 4297 feasible solutions (1110, 1042, 1075, 1070 from the I, II, III, IV nodes, respectively). The CPU time was approximately 8 h per node (platform: Intel Xeon 2.4 GHz, 2 GB RAM). After we combined 4297 feasible solutions, we obtained 326 Pareto optimal ones (84, 78, 87, 77 solutions from the I, II, III, IV nodes, respectively). Further investigation and solution of this problem can be carried out as in the example of the two-mass dynamical system (Section 3.1).

4. Conclusion

The *Parameter Space Investigation* (PSI) method plays a crucial role in engineering optimization in a sense that it *guides the correct statement and solution of many engineering problems*. This method has been widely integrated into various fields of industry, science, and technology [5]. The PSI method is implemented in the software package MOVI (which can be executed on a standard PC running MS Windows), a comprehensive software system that enables methodologically rigorous multicriteria analysis.

While working with applied problems of optimization (which are often ill-posed), it is often necessary to reconsider initial intervals of variation of design variables, functional and criteria constraints. Various elements of MOVI software package (such as test tables, histograms of feasible solutions, graphs of dependencies of criterion vs. criterion and criterion vs. design variable) allow an expert to (1) correctly construct the feasible solution set,

(2) analyze obtained results, and (3) make a decision about the most preferable solution(s) on the Pareto set. Such an analysis is necessary in order to obtain optimal solutions.

The ability to apply MOVI in a parallel mode can be used for multicriteria optimization of many large-scale systems *without decomposing their mathematical model into separate subsystems*. Given that many complex systems do not currently have known mathematical models, application of MOVI in a parallel mode can be extremely helpful. In particular, parallel computation can be used for the search for optimal solutions for automobiles, planes, ships, machine tools, problems of WWW, and other multicriteria problems with hundreds and thousands of design variables. Likewise, MOVI and PSI method can be used in conjunction with the traditional methods for analysis of large systems involving (1) decomposition of the system into subsystems, (2) optimization of subsystems, and (3) aggregation of optimal subsystems. The parallel mode in MOVI also allows to decrease the computation time and solve many problems that until recently appeared to be intractable.

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