

Visualization Approaches for the Prototype Improvement Problem

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ABSTRACT

One of the basic engineering optimization problems is improvement of the prototype. This problem is often encountered by industrial and academic organizations that produce and design various objects (e.g. motor vehicles, machine tools, ships, and aircrafts). This paper presents an approach for improving the prototype by constructing the feasible and Pareto sets while performing multicriteria analysis. We introduce visualization methods that facilitate construction of the feasible and Pareto sets. Using these techniques developed on the basis of Parameter Space Investigation method, an expert can correctly state and solve the problem under consideration in a series of dialogues with the computer. Finally, we present a case study of improving the ship prototype. Copyright © 2008 John Wiley & Sons, Ltd.

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1. INTRODUCTION

The prototype improvement (or operational development) problem can be defined as follows: There exists a real object (e.g. car, airplane, machine tool, or ship) that we call a ‘prototype’. The prototype is described by k performance criteria that take values $\Phi^p = (\Phi_1^p, \dots, \Phi_k^p)$. It is necessary to improve all or the most significant criteria of the prototype. The problem of operational development of a prototype is currently one of the most pressing and complex design problems. This problem is often encountered in the production of machine tools, cars, ships, and aircrafts, where significant resources are allocated for the operational development of the existing object. Furthermore, making the operational development of the prototype as quick as possible is highly desirable. Prior studies provide examples of real-life multicriteria problems of improving the prototype of a car, valve gear, metal-cutting machine tools, gear units, flexible manufacturing systems (Statnikov and Matusov, 1995; Gobbi

et al., 2000), space shuttle, nuclear reactor, unmanned vehicle configuration, airplane (Statnikov, 1999; Stadler and Dauer, 1992), ship (Anil, 2005; Statnikov and Matusov, 1995), truck frame, multi-stage axial flow compressor for an aircraft engine, robot, machine tools and their units, rear axle housing for a truck (Statnikov and Matusov, 2002), a parafoil-load delivery system (Yakimenko and Statnikov, 2005), a controllable descending system (Dobrokhodov *et al.*, 2003), vibration machines (Sobol’ and Statnikov, 2006), air bearing (Barrans and Bhat, 2003), and so on.

To solve the above problem, the prototype’s operational development requires two stages. In the *first stage*, based on the tests one must identify the mathematical model of the object and determine its parameters. To this end, one can solve an identification problem by working with particular adequacy (proximity) criteria. By adequacy criteria we mean the discrepancies between the experimental and computed data, the latter being determined on the basis of mathematical model. The number of adequacy criteria can reach many dozens if not hundreds. Multicriteria identification and adequacy of the mathematical models are discussed in detail in Statnikov and Matusov (2002). In the *second stage*, an expert formulates and solves the

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multicriteria optimization problem using the performance criteria and the mathematical model whose adequacy has been established in the first stage.

Sometimes we face another situation: The mathematical model of the prototype is given and we know the values of criteria of the prototype (Anil, 2005). In this case, we proceed with analysis that resembles the second stage of the operational development of the prototype.

The present work is devoted to approaches for improving the basic performance criteria by changing the design variable, functional, and criteria constraints while constructing the feasible and Pareto optimal solution sets. These approaches allow one to determine the potential of improving the prototype.

In general, the prototype improvement problem possesses several distinctive features outlined below. The methodology proposed in the present work is devoted to analysis of such problems.

1. The problem is essentially multicriteria, and the criteria are usually contradictory. For this reason there are difficulties in defining the criteria constraints correctly.
2. The initial constraints on the design variables, criteria, and functional dependencies may result in an empty or very sparse feasible solution set. Therefore, it is necessary to correct the initial statement of the problem. Thus, the formulation and solution of the problem have to comprise a single process (Sobol' and Statnikov, 2006; Statnikov and Matusov, 2002).
3. The feasible solution set can be multiply connected, and its volume may be several orders of magnitude smaller than that of the domain where the optimal solutions are sought. Generally, both the feasible solution set and the Pareto optimal set are nonconvex.
4. Mathematical models are usually complex systems of equations (including differential equations) that may be nonlinear, deterministic or stochastic, with distributed or lumped parameters. Information about the smoothness of goal functions is usually not available.
5. The analysis of Pareto optimal set to determine the most preferable solution does not pose a challenge to the expert (Ozernoy, 1988; Statnikov, 1999). This is because experts have a sufficiently well-defined system of preferences

(Statnikov and Matusov, 2002) and the Pareto optimal set often contains only a few solutions due to stringent constraints.

Therefore, constructing the feasible solution set is the most important step in formulating and solving the prototype improvement problems.

In the present paper, solution of the prototype improvement problem is based on construction of the feasible solution set via the *Parameter Space Investigation* (PSI) method that is widely used in various fields of industry, science, and technology (Sobol' and Statnikov, 2006; Statnikov and Matusov, 1995, 2002; Statnikov *et al.*, 2006). We introduce novel visualization procedures that guide experts in analysis of such problems. In some sense, the proposed visualization procedures are 'diagnostic tools' for the experts.

During the last 30 years, significant research has been done for visualization of two dimensional projections of criterion, design, and mixed vectors (e.g. see Cleveland, 1985; Lotov *et al.*, 2004; Meisel, 1973). It is also recognized in the field that these projections by themselves cannot give a comprehensive idea about localization of the feasible solution set in the multidimensional design variable and criteria spaces. The novelty of visualization techniques proposed in this work is that they are used together with the multicriteria test tables obtained by the PSI method during construction of the feasible solution set. These tools are an important addition to the PSI method but they cannot substitute for it. Thus, the main thesis of the present paper is that in order to construct the feasible solution set, it is necessary to consider both multicriteria test tables and various visualization tools.

This paper is organized as follows: Section 2 discusses the formulation and solution of the prototype improvement problem by optimizing performance criteria. In this section, we also briefly review the PSI method that allows to construct the feasible and Pareto sets. The visualization tools are discussed in Section 3. The geometric interpretation of the prototype improvement problem is provided in Section 4. Finally, to illustrate the usefulness of the techniques presented in this paper, Sections 5 and 6 contain a case study that solves the problem of improving the preterminal ship design prototype by using the PSI method and various visualization tools.

2. FORMULATION AND SOLUTION OF THE PROTOTYPE IMPROVEMENT PROBLEM

2.1. Generalized formulation of multicriteria optimization problems

We assume that a prototype depends on r design variables $\alpha_1, \dots, \alpha_r$ representing a point $\alpha = (\alpha_1, \dots, \alpha_r)$ in the r -dimensional space. In the general case, one has to take into account *design variable*, *functional*, and *criteria constraints*.

The *design variable constraints* have the form $\alpha_j^* \leq \alpha_j \leq \alpha_j^{**}$, $j = 1, \dots, r$. Constraints α_j^* and α_j^{**} define a parallelepiped Π in the r -dimensional design variable space. From the expert's perspective, the values of design variable constraints can be modified, if that leads to improvement of the basic criteria.

The *functional constraints* can be written as follows: $C_l^* \leq f_l(\alpha) \leq C_l^{**}$, $l = 1, \dots, t$, where $f_l(\alpha)$ is a functional relation, C_l^* and C_l^{**} are some constants. Functional relations together with constraints are some requirements of the object that sometimes an expert can successively revise in order to improve the basic performance criteria. These can be norms, standards, and other requirements such as mass, overall dimensions, allowable stress in structural elements, and so on.

The operation of a prototype is described by the particular performance criteria $\Phi_v(\alpha)$, $v = 1, \dots, k$. All other things being equal, it is desired that these criteria are optimized. For simplicity, we assume that functions $\Phi_v(\alpha)$ are to be minimized. To avoid situations in which the expert regards the values of some criteria as unacceptable, we introduce *criteria constraints* in the form $\Phi_v(\alpha) \leq \Phi_v^{**}$, $v = 1, \dots, k$, where Φ_v^{**} is the worst value of criterion $\Phi_v(\alpha)$ acceptable to an expert. These constraints are repeatedly revised during solution of the problem. The choice of Φ_v^{**} is discussed in the following sub-section.

The design variable, functional, and criteria constraints define the *feasible solution set* $D \subset \Pi$.

Let us formulate one of the basic problems of multicriteria optimization. It is necessary to define the feasible solution set D and find a set $P \subset D$ such that

$$\Phi(P) = \min_{\alpha \in D} \Phi(\alpha), \quad (1)$$

where $\Phi(\alpha) = (\Phi_1(\alpha), \dots, \Phi_k(\alpha))$ is the criterion vector and P is the *Pareto optimal set*. We mean

that $\Phi(\alpha) < \Phi(\beta)$ if for all $v = 1, \dots, k$, $\Phi_v(\alpha) \leq \Phi_v(\beta)$ and for at least one $v_0 \in \{1, \dots, k\}$, $\Phi_{v_0}(\alpha) < \Phi_{v_0}(\beta)$.

Finally, an expert determines a vector of design variables $\alpha^O \in P$ that is the most preferred among the vectors belonging to set P .

2.2. Parameter space investigation (PSI) method

Now we proceed by describing the PSI method that allows to determine Φ_v^{**} and, hence, the feasible solution set correctly. The PSI method is based on the investigation of the parallelepiped Π with points of uniformly distributed sequences (e.g. LP τ sequences), see Stadler and Dauer (1992), Statnikov and Matusov (1995, 2002) and Statnikov *et al.* (2006) for details. The method consists of three stages:

Stage 1. Compilation of test tables via computer. First, one chooses N test points $\alpha^1, \dots, \alpha^N$ that satisfy the functional constraints. Then all the particular criteria $\Phi_v(\alpha^i)$ are calculated at each of the points α^i ; for each of the criteria a test table is compiled so that the values of $\Phi_v(\alpha^1), \dots, \Phi_v(\alpha^N)$ are arranged in increasing order, i.e.

$$\Phi_v(\alpha^{i_1}) \leq \Phi_v(\alpha^{i_2}) \leq \dots \leq \Phi_v(\alpha^{i_N}), \quad v = 1, \dots, k \quad (2)$$

where i_1, i_2, \dots, i_N are the numbers of tests (a separate set for each v). Taken together, the k tables form a complete test table.

Stage 2. Preliminary selection of criterion constraints. This stage includes interaction with an expert. By analysing inequalities (2), an expert specifies the criteria constraints Φ_v^{**} . Actually, an expert has to consider one criterion at a time and specify the respective constraints. One analyses a test table and imposes the criterion constraint. Then one proceeds to the next table, and so on. Note that the revision of the criteria constraints within the limits of the test tables does not lead to any difficulties for an expert.

Since we want to minimize all criteria, Φ_v^{**} should be the maximum values of the criteria $\Phi_v(\alpha)$, which guarantee an acceptable level of the object's operation. If the selected values of Φ_v^{**} are not a maximum, then many interesting solutions may be lost, since some of the criteria are contradictory. Moreover in some cases the feasible solution set may be empty.

In practice, an expert imposes the criteria constraints in order to improve a prototype by all criteria simultaneously. If it is impossible, one improves a prototype by the most important criteria. In process of dialogues with computer, an expert repeatedly revises criteria constraints

and carries out the multicriteria analysis. The PSI method gives expert valuable information on the advisability of revising various criteria constraints with the aim of improving the basic criteria. The expert sees what price he pays for making concessions in various criteria, i.e. what one loses and what one gains.

Stage 3. Solvability of problem (1) via computer. Let us fix a criterion, say $\Phi_{v_1}(\alpha)$, and consider the corresponding test table (2). Let S_1 be the number of the values in the table satisfying the selected criterion constraint:

$$\Phi_{v_1}(\alpha^{i_1}) \leq \dots \leq \Phi_{v_1}(\alpha^{i_{S_1}}) \leq \Phi_{v_1}^{**} \quad (3)$$

Then criterion Φ_{v_2} is selected by analogy with Φ_{v_1} and the values of $\Phi_{v_2}(\alpha^{i_1}), \dots, \Phi_{v_2}(\alpha^{i_{S_1}})$ in the test table are considered. Let the table contain $S_2 \leq S_1$ values such that $\Phi_{v_2}(\alpha^{i_j}) \leq \Phi_{v_2}^{**}$, where $1 \leq j \leq S_2$. Similar procedures are carried out for each criterion. Then if at least one point can be found for which all criteria constraints are valid simultaneously, then the set D is nonempty and problem (1) is solvable. Otherwise, an expert should return to Stage 2 and make certain concessions in the specification of Φ_v^{**} . However, if the concessions are highly undesirable, then one may return to Stage 1 and increase the number of points in order to repeat Stages 2 and 3 using extended test tables (Statnikov and Matusov, 2002).

The procedure is to be iterated until D is nonempty. Then the Pareto optimal set is constructed in accordance with the definition presented above. This is done by removing those feasible points that can be improved with respect to all criteria simultaneously.

Thus, in accordance with the PSI method, the criteria constraints are determined in the dialogue of an expert with a computer. Then an expert should determine the Pareto set P and after analysing P find the most preferred solution $\Phi(\alpha^O)$ surpassing the prototype in all criteria, or at least the most important ones. For the class of problems considered in this paper, experts do not encounter serious difficulties in analysing the Pareto optimal set and in choosing the most preferred solution. This is because experts have a sufficiently well-defined system of preferences for this type of problems (Statnikov and Matusov, 2002) and the Pareto optimal set often contains only a few solutions due to stringent constraints. More complex cases of the decision making, where preferences on the Pareto optimal set are not

necessarily stable are discussed in Lichtenstein and Slovic (2006), Wu and Azarm (2001) and Zitzler *et al.* (2003).

2.3. Pseudo-criteria

As mentioned above, often an expert cannot determine the functional constraints correctly. For example, in practical problems 'good' solutions may lie beyond the limits imposed by the constraints.¹ If informed of this, an expert may be ready to modify the constraints so that the 'good' solutions will belong to the feasible solution set. Below we present an approach to obtain such information.

Instead of the function $f_l(\alpha) \leq C_l^{**}$, $l = 1, \dots, t$ with the *soft* constraint C_l^{**} , we introduce an additional criterion² $\Phi_{k+l}(\alpha) = f_l(\alpha)$, which we call a *pseudo-criterion*. To find the value of the constraint Φ_{k+l}^{**} one has to compile a test table containing $\Phi_{k+l}(\alpha)$. By using the aforementioned algorithm, one can define Φ_{k+l}^{**} in a way that prevents the loss of interesting solutions.

In general, when solving a problem with soft functional constraints, one has to find the set D , taking all performance criteria and pseudo-criteria into account. In other words, one must solve the problem with the constraints

$$\Phi_v(\alpha) \leq \Phi_v^{**}, \quad v = 1, \dots, k, k+1, \dots, k+t$$

Thus, to define the feasible solution set, we consider a multicriteria problem with $k+t$ criteria. Notice, however, that the pseudo-criteria are not considered when constructing the Pareto optimal set.

It is worthwhile to mention that many single-criterion problems may have soft functional constraints as well. In these cases the definition of the feasible solution set is also very important. For the definition of the feasible solution set, it is necessary to represent functional dependencies (with soft constraints) as pseudo-criteria. In other words, we have to consider such single-criterion problems as multicriteria ones.

¹If these constraints are given unreasonably rigid, solutions that do not satisfy them are not considered at all. For this reason the feasible solution set can be poor or even empty.

²Recall that we have test tables that contain *performance criteria* $\Phi_v(\alpha)$, $v = 1, \dots, k$.

2.4. Construction of the combined feasible and Pareto sets

Quite often the analysis of test tables points to advisability of correcting the boundaries of the initial parallelepiped (i.e. re-defining the constraints on the design variables) and constructing a new parallelepiped. Suppose that appropriate investigations have been performed in the new parallelepiped, and that the corresponding feasible set has been constructed. Now it is necessary to combine the feasible sets constructed in both these parallelepipeds and define the Pareto optimal solutions in the combined feasible set. The procedure of the repeated correction of parallelepipeds and construction of the combined feasible and Pareto optimal sets is essential for our problem, as emphasized in Section 4. Constructing the combined Pareto optimal set allows to estimate the contribution of each parallelepiped to this set and the expediency of correcting the initial problem (see Section 6).

There is another situation where construction of the combined feasible and Pareto sets is needed. These are the problems where calculation of the criteria vector for one test requires a significant amount of computer time (e.g. see Section 2.6). Similarly, these are problems that require to perform a very large number of tests. For the efficient solution of such problems, the desired number of tests N can be distributed among k computers and each computer will perform a search for the feasible solutions for its own subproblem by conducting $\sim N/k$ tests. Next, the obtained feasible solution sets are combined and the Pareto optimal solution set is constructed (Statnikov *et al.*, 2005a, 2006).

2.5. Number generators for systematic search in the design variable space

To investigate design variable space, we use *uniformly distributed sequences*. At present, the so-called $LP\tau$ sequences are among the best ones in terms of uniformity characteristics (Sobol' and Statnikov, 2006; Statnikov and Matusov, 1995, 1996, 2002). These sequences are used to compute N test points $\alpha^1, \dots, \alpha^N$ in the design variable space during Stage 1 of the PSI method. Other uniformly distributed sequences and nets (Faure, 1982; Halton, 1960; Hammersley, 1960; Hlawka and Taschner, 1991; Kuipers and Niederreiter, 1974; Niederreiter, 1990; Statnikov and Matusov, 1996) can be also successfully used in the PSI method. However, prior to solving a specific problem, one

cannot say with certainty which uniformly distributed sequences are the most suitable. Much depends on the behaviour of the criteria, the form of the functional and design variable constraints, the number of tests, and the geometry of the feasible solution set (Statnikov and Matusov, 2002). Steuer and Sun (1995) have indicated an opportunity of using random number generators in the PSI method. Based on these recommendations, we have successfully applied random number generators in the PSI method to solve multicriteria problems with very high-dimensional design variable vectors (dimensionality = 1000) and to also solve the problems in the parallel mode (Statnikov *et al.*, 2005a, 2006). The experiments with various random number generators in the PSI method are described in Statnikov *et al.* (2005b).

2.6. Applications of the PSI method

The PSI method is implemented in the MOVI (*Multicriteria Optimization and Vector Identification*) software system (Yanushkevich *et al.*, 2004). The software package MOVI allows solution of problems where the number of design variables and criteria is not practically limited. Below we summarize several problems with different characteristics where the PSI method and MOVI software system have been successfully applied. These examples also demonstrate that the number of tests depends on constraints, dimensionality of design variable vector, and time of calculating one criteria vector for given design variable values (Statnikov *et al.*, 2006).

(A) *Problem of naval ship design* (Anil, 2005). Among the particular features of this problem are the high dimensionality of the design variable vector (45 design variables) and the difficulties in improving a reasonably good prototype under strong constraints on six performance criteria (propulsion power factor, electrical power factor, volume factor, region factor, weight factor, and cost), nine pseudo-criteria, and seven functional dependences. Since calculation of one vector of criteria took < 1 s and the design vector is of high dimensionality, 200 000 tests were conducted in the first experiment. Multicriteria analysis showed the necessity of repeated correction of the constraints, and because of this, five more experiments with 200 000–500 000 tests have been performed. Each subsequent experiment was carried out on the basis of the previous one. In the first two experiments no feasible solutions were found; in

the third experiment a few feasible solutions were obtained; and it was only in the fourth experiment where some satisfactory results were found. An analysis of these results allowed to define a region of 'good' solutions where subsequent experiments were carried out. As a result, 26 Pareto optimal solutions surpassing the prototype in all criteria were identified.

(B) *Problem of design of a car for shock protection* (Statnikov and Matusov, 2002). Unlike the previous problem where calculation of a criteria vector took <1 s, the criteria in this problem are based on finite element model with thousands of elements and nodes, and it takes several hours to compute one vector of criteria. This problem has 10 criteria: the mass of structure and residual strains in the car body after impact in the nine most dangerous points of the rear panel. The number of design variables in this problem is 13, that at face value suggests a large computational experiment. However, since it requires such a large amount of computational time to compute one vector of criteria, the optimization of design variables is very difficult to implement. Therefore, the initial model was decomposed in two approximate models of bumper and rear panel. It was necessary to define the consistent solutions between subsystems of bumper and rear panel. For this purpose, 300 tests were carried out. Each test required no more than 10 min. The analysis of the obtained solutions has shown that the prototype cannot be improved. As a result, the problem has been reformulated (i.e. the designs of the bumper and the rear panel have been modified by introducing additional stiffening ribs). Fifteen consistent design variable vectors were defined. For these vectors we have calculated all criteria using the original model. The number of feasible solutions satisfying all constraints of the structure was nine. The number of Pareto optimal solutions was five. This approach allowed to improve the prototype in the successive application of the PSI method.

(C) *Problem of operation development of a truck* (Statnikov and Matusov, 2002). In terms of computational time, this problem falls in-between two problems described above. The computation of a vector of criteria based on a system of complex differential equations took ~ 3 min. In the identification phase of this problem, we identified 16 parameters of the mathematical model using 65 adequacy criteria and defined to what extent the model corresponds to the real

system. In the initial problem statement, 4096 tests were conducted. Only seven solutions met the criteria constraints and thus entered the feasible solution set. After analysing obtained results, new boundaries of design variables were defined. The same number of tests (4096) was conducted. As a result, 11 more feasible solutions were obtained. On the basis of solution of the identification problem and definition of the feasible solution set, the problem of optimization by 20 performance criteria was solved next. These criteria were divided into the following groups: (1) comfort, (2) durability, (3) load preservation, and (4) safety. Twenty parameters were varied. Optimization was aimed to improve the prototype. 4096 tests were conducted and 21 solutions satisfied all constraints and entered the feasible solution set. The Pareto set consisted of 20 solutions.

3. TOOLS FOR VISUALIZATION

Below we describe a few visualization tools that are particularly useful for multicriteria analysis.³ It is important to emphasize that these tools should be used together with the test tables (see Section 2.2) that allow to define feasible solutions in the problems of *any dimensionality*. All the tools listed below are implemented in the software system MOVI.

- *Histograms* of the distribution of feasible and Pareto optimal solutions. The intervals $[\alpha_j^*, \alpha_j^{**}]$, $j = 1, \dots, r$ are divided into 10 identical sub-intervals. Above each subinterval, the number of feasible designs entering this subinterval is indicated. Analysing the histograms and graphs reveals how the feasible and Pareto sets are distributed in design variable space. The histograms play the main role in correcting design variable and other constraints.
- *Graphs: 'criterion vs. design variable.'* We consider projections of the *multidimensional points* $\Phi_v(\alpha^i)$, $v = 1, \dots, k$, $i = 1, \dots, N1$ onto the plane

³In this paper we indicate only some basic tools. A detailed description of other tools, e.g. tables of functional and criteria failures, tables of criteria, tables of design variables, graphs of criteria vs. design variables for the Pareto optimal solutions is provided in Statnikov *et al.* (2005a, 2006) and Yanushkevich *et al.* (2004).

$\Phi_v \alpha_j$. These projections provide information about the sensitivity of criteria to the design variables, and also point to localization of the feasible solution set. Significance of the sensitivity of criteria to the design variables is indicated in (Sobol', 2001).

- *Graphs: 'criterion vs. criterion.'* After N tests, $N1$ design variable vectors have entered the test table. We consider projections of the *multi-dimensional points* $\Phi_v(\alpha^i)$, $v = 1, \dots, k$, $i = 1, \dots, N1$ onto the plane $\Phi_i \Phi_j$. These projections provide information about dependencies between criteria and localization of the feasible solution and Pareto optimal sets in criteria space.

We note that the graphs 'criterion vs. criterion' by themselves are not sufficiently informative, and it is indeed difficult or sometimes impossible to make any conclusions solely based on them. That is why we propose here a generalized approach of solution of the prototype improvement problem. Specifically, test tables and histograms together with graphs give a comprehensive idea about the feasible solution set and necessity of correcting initial problem statement. This approach allows an expert to (1) estimate the effect of all constraints, (2) determine significant design variables, and (3) compare the values of the criteria of a prototype with the results obtained by the PSI method. The examples of graphs and histograms are provided in Section 6.

4. GEOMETRIC INTERPRETATION OF IMPROVING A PROTOTYPE PROBLEM

Below we consider the basic scenarios for improving a prototype by using the PSI method and provide guidelines to the experts.

Case 1a. The values of design variable vector α^p and criteria vector $\Phi^p = (\Phi_1^p, \Phi_2^p)$ of the prototype are known, see Figure 1. The boundaries of the initial parallelepiped Π_1 are defined as admissible deviations of the design variables from the corresponding values of the prototype α^p . In Figure 1(b), vector α^p is located in the centre of the parallelepiped Π_1 . Here and henceforth the parallelepiped Π_1 corresponds to the initial statement of the optimization problem. On the basis of the PSI method, the feasible set D_Φ and the Pareto set P_Φ are defined (Statnikov and Matusov, 1995, 1996, 2002). The disconnected

feasible set is shaded in Figure 1. After analysing the Pareto set P_Φ , an expert determines the most preferable solution $\Phi^O \in P_\Phi$. Notice that D_α , P_α , α^p , and α^O are inverse images of D_Φ , P_Φ , Φ^p , and Φ^O , respectively, in the design variable space.

More general scenarios are discussed below. Next we will consider the situations when an expert should change the initial parallelepiped Π_1 .

Case 1b. Similarly to case 1a, we assume that the values of the vector α^p and vector Φ^p are known, see Figure 2. Let the feasible set, the Pareto set P_{Φ_1} (curve AB), and the most preferable solution Φ^{O1} be defined in the initial statement of the problem, see Figure 2(a).

In many optimization problems some coordinates of the optimal solution $\alpha^{O1} = (\alpha_1^{O1}, \dots, \alpha_r^{O1})$ are located in the vicinity of the borders of the initial parallelepiped Π_1 . In these cases, it is logical to change the initial borders and to perform new investigations. A parallelepiped Π_2 with new boundaries is shown in Figure 2(b). As a result of investigating Π_2 , the new feasible set, the Pareto set P_{Φ_2} (curve CD), and the most preferable solution Φ^{O2} are defined, Figure 2(a). Since the feasible sets

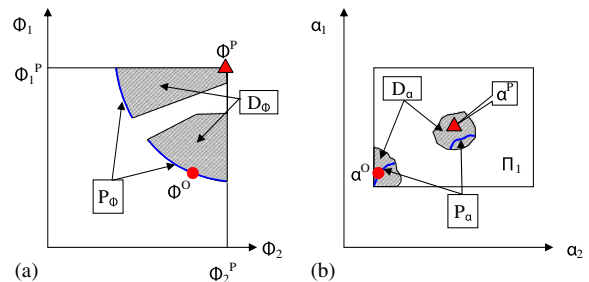


Figure 1. Improving a prototype Φ^p . Φ^O is the most preferable solution.

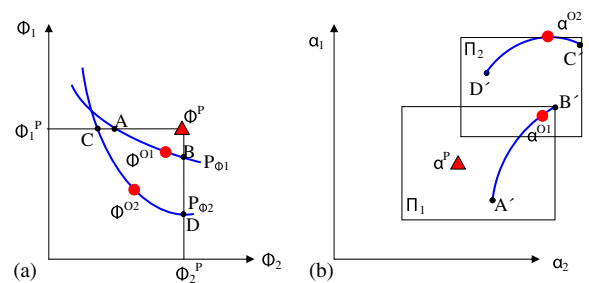


Figure 2. Improving the Pareto optimal solution Φ^{O1} and definition of the most preferable solution Φ^{O2} , $\alpha^{O1} \in \Pi_1$, $\alpha^{O2} \in \Pi_2$.

were defined in Π_1 and Π_2 , an expert should construct the combined feasible and Pareto optimal sets. In this example the combined Pareto set corresponds to P_{Φ_2} , Figure 2(a). In other words, all the solutions belonging to P_{Φ_2} are better than the solutions belonging to P_{Φ_1} . Notice that $A'B'$, $C'D'$, α^{O1} , α^{O2} , and α^p are inverse images of AB , CD , Φ^{O1} , Φ^{O2} , and Φ^p , respectively, in the design variable space (Figure 2b).

On the basis of the PSI method, an expert can determine the most significant design variables. If an expert prefers to vary only the significant design variables, the dimensionality of the new parallelepiped Π_2 can be smaller than the dimensionality of the initial parallelepiped Π_1 .

Case 1c. In contrast to the case 1b, the new combined Pareto set (curve AEC) consists of two curves, $AE \in P_{\Phi_1}$ and $EC \in P_{\Phi_2}$, see Figure 3(a). Here D_Φ is a combined feasible set. Notice that $A'B'$, $C'D'$, and $A'E/C'$ are inverse images of AB , CD , and AEC in the design variable space respectively (Figure 3b). After analysing the combined Pareto set, an expert determines the most preferable solution Φ^O . Notice $\Phi^O \in DC$. From Figure 3(b) it follows that $\alpha^O \in \Pi_2$. The example illustrating this case is presented in Sections 5 and 6.

Case 1d. The value of Φ^p is known, but many values of the design variables of a prototype can be unknown, see Figure 4. This case is common in the first stage of the prototype's operational development. In this stage, an expert should identify a mathematical model and its parameters using adequacy criteria but usually has only a rough idea about the limits of many identified design variables. Assume that an expert cannot identify the vector α^p in Π_1 and the feasible set is empty, see Figure 4(b).

The parallelepiped Π_1 must be corrected and an expert constructs a new parallelepiped Π_2 where he can identify the vector α^p . Then an expert states and solves the problem of improving a prototype by the performance criteria (the second stage mentioned in the Introduction). In an attempt to improve the prototype, an expert constructs and investigates parallelepipeds $\Pi_3, \dots, \Pi_{F-1}, \Pi_F$. In Figure 4(b) the feasible set D_α and Pareto set P_α are defined in the parallelepiped Π_F . The feasible set D_α is shaded. An expert determines the most preferable solution Φ^O in the Pareto optimal set P_Φ , Figure 4(a). Notice that P_α , α^p , and α^O are inverse images of P_Φ , Φ^p , and Φ^O in the design variable space respectively (Figure 4b).

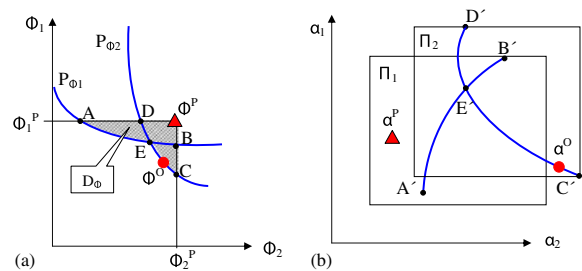


Figure 3. Construction of the combined Pareto optimal set AEC; $\alpha^p \in \Pi_1$, $\alpha^O \in \Pi_2$.

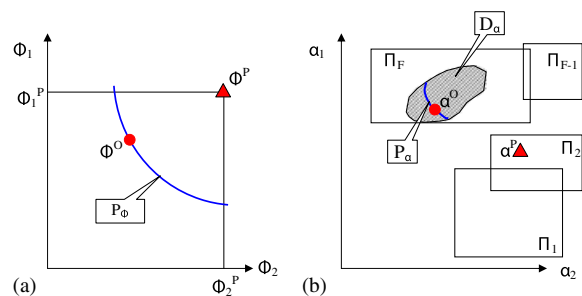


Figure 4. Identification of the vector α^p and definition of the most preferable solution Φ^O ; $\alpha^p \in \Pi_2$, $\alpha^O \in \Pi_F$.

In general, an expert corrects the statement of the problem by constructing the parallelepipeds $\Pi_1, \dots, \Pi_{F-1}, \Pi_F$, and then constructs the combined feasible and Pareto sets. These steps are carried out by using the PSI method. For more details (see Statnikov *et al.*, 2005a).

In the above cases, the obtained Pareto optimal solution Φ^O surpassed the prototype Φ^p in all criteria. Situations in which an expert cannot improve all the criteria simultaneously are shown in cases 2 and 3 below.

Case 2. A sufficiently typical situation is shown in Figure 5. After constructing the Pareto optimal set P_Φ an expert revealed that $\Phi^p \in P_\Phi$. The feasible set D_Φ is shaded in Figure 5. The multicriteria analysis of the obtained Pareto optimal solutions allowed to determine the most preferable solution Φ^O .

Case 3. Here an expert wishes to improve the prototype, and *a priori* he defines the value of the criteria vector as $\Phi^W = (\Phi_1^W, \dots, \Phi_k^W)$, see Figure 6(a). However in contrast to the former cases, here all or some values $\Phi_1^W, \dots, \Phi_k^W$ can be unattainable. Assume that the investigation of the initial problem's statement revealed that the

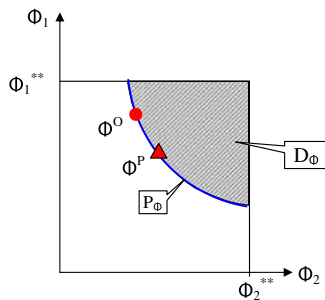


Figure 5. $\Phi^P \in P_{\Phi}$. Φ^O is the most preferable solution.

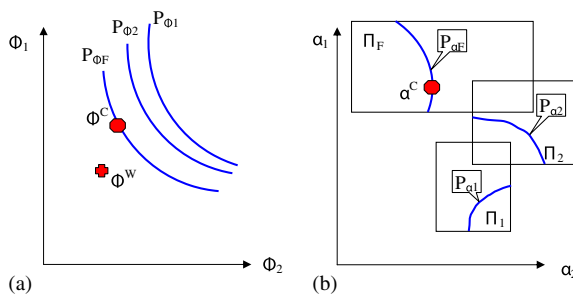


Figure 6. Pareto optimal set $P_{\Phi F}$ cannot ‘reach’ the solution Φ^W . Φ^W is unattainable, α^W does not exist. Solution Φ^C is a compromise.

feasible set is empty and an expert decides to correct the initial parallelepiped Π_1 . He corrects the statement of the problem repeatedly, including the construction of the parallelepipeds Π_2, \dots, Π_F , as shown in Figure 6(b). The Pareto sets $P_{\Phi 1}, P_{\Phi 2}, \dots, P_{\Phi F}$ were defined after the construction and investigation of the above parallelepipeds, respectively, Figure 6(a). In our case, despite the corrections of the design variable and other constraints, the feasible solution set remains empty.⁴ Thus, the Pareto set does not contain any feasible solutions. Therefore, the desired values of local criteria Φ_v^W , $v = 1, \dots, k$ are *unattainable*.

We offer two suggestions in this situation: First, an expert can make concessions and accept a compromise solution Φ^C , taking into account the most important criteria, Figure 6(a). ($P_{\alpha 1}, P_{\alpha 2}, \dots, P_{\alpha F}$, α^C are inverse images of $P_{\Phi 1}, P_{\Phi 2}, \dots, P_{\Phi F}$, Φ^C , respectively, in the design

variable space, Figure 6(b); α^W does not exist.) The second option is not to accept a compromise solution. In this case, it is possible that the expert’s wishes can be realized by creating a new object, which is different from the prototype. Therefore, the multicriteria analysis is helpful to answer the important question of how to improve the prototype and by how much.

The identification of mathematical models of different objects (vehicles, machine tools, and their units) and their improvement were described in detail in Statnikov and Matusov (2002). These analyses correspond to cases 1d–3. Similar problems concerning the parafoil-load delivery system and the controllable descending system were discussed in Dobrokhodov *et al.* (2003) and Yakimenko and Statnikov (2005).

Finally, we would like to mention that improvement of the prototype depends on (1) constructive scheme (topology of the object), (2) dimensionality of the design variable vector, significant design variables, range of their change, (3) materials constituting the design variables (physical and chemical properties).

5. EXAMPLE: THE PROBLEM OF IMPROVING A PRELIMINARY SHIP DESIGN PROTOTYPE

The mathematical model described below is based on the references (Bales, 1980; Fung, 1991). More details are also provided in Anil (2005). Since the example is used primarily to illustrate application of the proposed methodology, we do not emphasize specifics of the underlying physical problem. The problem has eight criteria that are defined implicitly. Two of the eight are performance criteria: Resistance Performance defined using Fung’s Resistance Prediction Algorithm (Bare Hull Residuary Resistance Coefficient) and the Seakeeping Performance defined by Bales Formula (Bales Seakeeping Rank). The other six criteria are pseudo-criteria. Since we have a readily available mathematical model, we need only to conduct the second stage of the operational development as described in the ‘Introduction.’

The model is based on 14 design variables:

- α_1 : length of design waterline (assumed to be equal to length between perpendiculars), m
- α_2 : beam of design waterline (assumed to be equal to beam amidships), m

⁴Earlier we considered cases in which feasible set D was nonempty and the Pareto set $P \subset D$.

- α_3 : draft (assumed to be equal to draft amidships), m
- α_4 : distance from the station 0 (FP) to the cut-up point, m
- α_5 : waterplane area forward of amidships, m^2
- α_6 : waterplane area aft of amidships, m^2
- α_7 : displaced volume forward of amidships, m^3
- α_8 : displaced volume aft of amidships, m^3
- α_9 : prismatic coefficient
- α_{10} : projected transom area, m^2
- α_{11} : projected transom width, m
- α_{12} : projected transom depth, m
- α_{13} : half entrance angle, degrees
- α_{14} : longitudinal centre of buoyancy from the fore perpendicular, m

Thus, the design variable vector is $\alpha = (\alpha_1, \dots, \alpha_{14})$. Values of the prototype design variables α^p and initial design variable constraints (i.e. parallelepiped Π_1) are provided in Table I.

We want to maximize the performance criterion Φ_1 (Seakeeping Rank,) and to minimize the performance criterion Φ_2 (Residuary Resistance Coefficient). The vector of performance criteria of prototype is $\Phi^p = (8.608; 1.968)$, see Table II.

There are 10 functional relations:

- $f1$: displacement, metric ton
- $f2$: block coefficient
- $f3$: maximum section area coefficient (assumed to be equal to midship section coefficient C_m)
- $f4$: waterplane area coefficient

- $f5$: waterplane area coefficient forward of amidships
- $f6$: waterplane area coefficient aft of amidships
- $f7$: draft-to-length ratio
- $f8$: cut-up ratio
- $f9$: vertical prismatic coefficient forward of amidships
- $f10$: vertical prismatic coefficient aft of amidships

The following rigid constraints are imposed on the above four functional relations:

- $f1 \leq 2100$
- $f2 \leq 0.51$
- $f3 \geq 0.77$
- $f4 \leq 0.84$
- $f4 \geq 0.80$

The others functional constraints $f5, \dots, f10$ are nonrigid, i.e. they can change in some limits, however it is difficult to formulate these constraints *a priori*. According to the PSI method the nonrigid functional relations should be interpreted as the *pseudo-criteria* i.e. $\Phi_3 = f5$, $\Phi_4 = f6$, $\Phi_5 = f7$, $\Phi_6 = f8$, $\Phi_7 = f9$, and $\Phi_8 = f10$, see Section 2.3. In this case, the constraints are defined during solution of the problem (on the basis of the analysis of the test table). Values $\Phi_3, \Phi_4, \Phi_5, \Phi_6, \Phi_7, \Phi_8$ for the prototype are shown in Table II. Therefore, to determine the feasible solution set we solve the

Table I. Table of design variables. Parallelepiped Π_1 .

Design variables	Prototype α^p	Π_1		Pareto optimal solutions in initial statement		
		Lower bound	Upper bound	α^{26087}	α^{75527}	α^{81087}
α_1	90.700	85.700	95.700	94.749	94.485	95.592
α_2	12.670	10.670	14.670	11.659	11.860	11.656
α_3	3.700	3.500	3.900	3.581	3.578	3.679
α_4	51.650	48.650	54.650	52.081	50.611	52.584
α_5	380.10	330.100	430.100	416.938	427.530	404.837
α_6	552.900	502.900	602.900	506.413	505.068	514.845
α_7	991.200	891.200	1091.200	930.403	952.688	940.701
α_8	1040.100	940.000	1140.000	1041.730	1066.618	1096.428
α_9	0.626	0.620	0.635	0.626	0.624	0.625
α_{10}	11.740	9.740	13.740	11.758	10.099	10.416
α_{11}	12.120	10.120	14.120	10.555	10.982	10.622
α_{12}	0.950	0.750	1.050	0.880	1.025	0.754
α_{13}	13.000	12.000	14.000	12.011	13.986	12.607
α_{14}	45.900	43.900	47.900	47.810	47.604	44.024

Table II. Table of criteria

Criteria	Prototype Φ^p	Pareto optimal solutions in initial statement			Pareto optimal solutions in second statement		
		#26087	#75527	#81087	# 74223	#49109	#106467
$\Phi_1(\max)$	8.608	14.342	14.267	12.614	14.537	15.4598	15.4591
$\Phi_2(\min)$	1.968	1.847	1.803	1.711	1.694	1.873	1.852
$\Phi_3(\text{pseudo})$	0.662	0.755	0.763	0.727	0.753	0.767	0.767
$\Phi_4(\text{pseudo})$	0.962	0.917	0.901	0.924	0.905	0.911	0.911
$\Phi_5(\text{pseudo})$	0.041	0.038	0.038	0.038	0.038	0.039	0.038
$\Phi_6(\text{pseudo})$	0.569	0.550	0.536	0.550	0.554	0.565	0.554
$\Phi_7(\text{pseudo})$	0.705	0.623	0.623	0.632	0.605	0.596	0.603
$\Phi_8(\text{pseudo})$	0.508	0.574	0.590	0.579	0.578	0.560	0.560
Criteria	Pareto optimal solutions in final statement						
	#113487	#4145	#68410	#39801	#53988	#72461	#75110
$\Phi_1(\max)$	15.308	15.183	15.131	15.052	14.887	14.805	14.601
$\Phi_2(\min)$	1.677	1.674	1.657	1.645	1.644	1.629	1.622
$\Phi_3(\text{pseudo})$	0.760	0.755	0.760	0.753	0.758	0.752	0.752
$\Phi_4(\text{pseudo})$	0.918	0.923	0.916	0.926	0.921	0.926	0.928
$\Phi_5(\text{pseudo})$	0.038	0.038	0.038	0.038	0.038	0.038	0.038
$\Phi_6(\text{pseudo})$	0.538	0.547	0.558	0.534	0.551	0.538	0.554
$\Phi_7(\text{pseudo})$	0.598	0.601	0.602	0.597	0.607	0.606	0.615
$\Phi_8(\text{pseudo})$	0.561	0.553	0.561	0.557	0.559	0.563	0.568

problem with *criteria vector* $\Phi = (\Phi_1, \Phi_2, \Phi_3, \Phi_4, \Phi_5, \Phi_6, \Phi_7, \Phi_8)$. The vector of functional relations is $(f1, f2, f3, f4)$.

In summary, we want to investigate the problem with a 14-dimensional design variable space and an 8-dimensional criteria space, keeping in mind complex constraints, which we need to correct in order to construct feasible solutions.

The analysis process summarized in the next section is based on application of the PSI method with LP_t sequences.

6. MULTICRITERIA ANALYSIS OF THE SHIP DESIGN PROTOTYPE IMPROVEMENT PROBLEM

6.1. Solution of the initial statement of the optimization problem

We performed $N = 131\,072$ tests in parallelepiped Π_1 and only $N_1 = 1487$ vectors entered the test table. The 129 585 solutions did not satisfy the functional constraints. Since we are solving the problem of improving the prototype, the performance criteria constraints equal to the values of the prototype ($\Phi_1^p = \Phi_1^{**} = 8.608$ and $\Phi_2^p = \Phi_2^{**} = 1.968$) were accepted. The constraints on pseudo-criteria were defined on the basis of

analysing test tables. As a result, $ND = 240$ vectors (including the prototype) entered the *feasible set*, and $NP = 3$ vectors are the *Pareto optimal solution set* (The remaining 1247 vectors did not satisfy the criteria constraints). The coefficient of the efficiency of searching the feasible solution set is $\gamma_F = 240/131\,072 = 0.0018$ (for Pareto optimal set it is $\gamma_P = 0.00002$). Very low values of coefficient γ_F point out the difficulties in searching the feasible solutions.

These vectors in the criteria space and design variable space are denoted as $\Phi^{26087}, \Phi^{81087}, \Phi^{75527}$ and $\alpha^{26087}, \alpha^{81087}, \alpha^{75527}$ respectively. To simplify the notation, we will denote vectors of criteria corresponding to the i th test (i.e. Φ^i) simply as # i . Therefore, $\Phi^{26087}, \Phi^{81087}, \Phi^{75527}$ are written as # 26087, # 81087, and # 75527 (see Table II). The values of design variables and criteria of Pareto optimal solutions are given in Tables I and II.

Below we briefly show some visualization tools that led to improvement of the present results.

Graphs. 'criterion vs. criterion.' Figure 7 shows dependency between the first criterion (Seakeeping Rank) and the second criterion (Residuary Resistance Coefficient).

Example of dependencies between the criterion Φ_1 and the pseudo-criterion Φ_3 is shown in Figure 8.

Graph. 'criterion vs. design variable.' Figure 9 shows the dependency of the criterion Φ_2 vs. the design variable α_{10} .

Histograms of feasible solutions. The distributions of feasible solutions for the range of the 2nd and 10th design variables in the initial parallelepiped Π_1 are shown in Figure 10(a). We can see that there are large 'gaps' that do not contain any feasible solutions. Similar distributions with 'gaps' are observed for the 5th, 6th, and 9th design variables.

6.2. Second statement and solution of the optimization problem. Parallelepiped Π_2

The analysis of the feasible solutions has pointed ways for correction of the initial problem statement. The reasoning for construction of the parallelepiped Π_2 is given below:

- From the histogram of the second design variable (Figure 10a), it follows that the value of the upper boundary can be decreased to $\alpha_2^{**} = 13.5$ (recall that in the initial parallelepiped Π_1 we had $\alpha_2^{**} = 14.67$).
- From the graph 'Criterion vs. Design Variable' (Figure 9) and the histogram of the 10th design

variable (Figure 10a), it follows that the value of the upper boundary can be decreased to $\alpha_{10}^{**} = 13.0$ (recall that in the initial parallelepiped Π_1 we had $\alpha_{10}^{**} = 13.74$).

- The other boundaries of the parallelepiped Π_2 were determined similarly. In the process of the definition of boundaries of parallelepiped Π_2 , the values of the prototype design variables and Pareto optimal solutions # 26087, # 75527 and # 81087 were also considered.

Again, $N = 131\,072$ tests were conducted. $N1 = 18\,270$ vectors entered the test table. The pseudo-criteria constraints were defined based on the analysis of the test tables, while constraints on performance criteria were kept unchanged. This time $ND = 17\,302$ vectors (including the prototype) entered the *feasible solution set*, and $NP = 14$ *Pareto optimal solutions* were identified (# 87511, # 21855, # 63919, # 49109, # 106467, # 90819, # 78907, # 116871, # 31819, # 64407, # 74223, # 80159, # 105823, and # 22671). It follows that the coefficient of the efficiency of searching the feasible solutions (γ_F) was increased more than 70 times. Furthermore, the histograms in Π_2 have much better distributions of the feasible solutions than in Π_1 (see Figure 10).

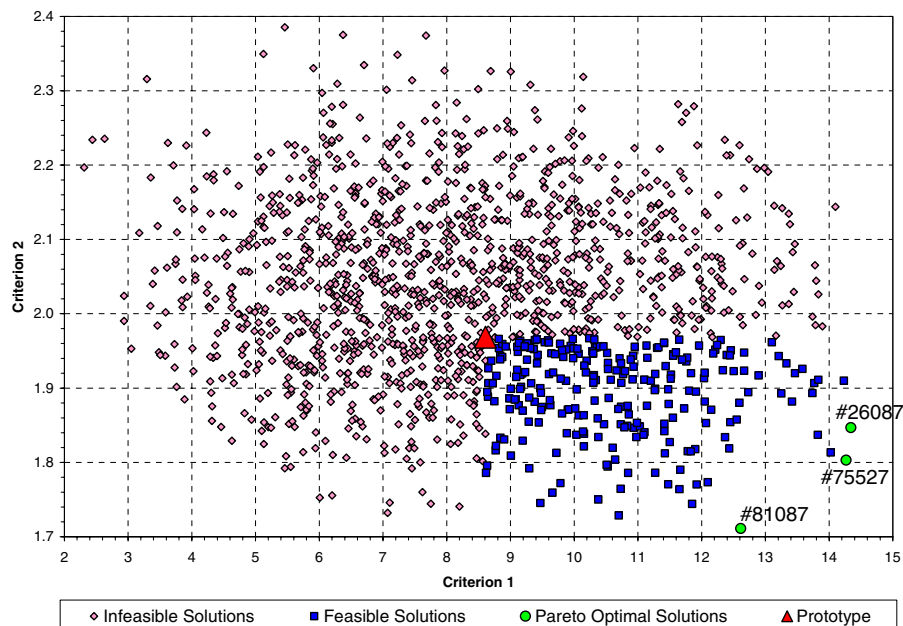


Figure 7. The dependency between criteria Φ_1 and Φ_2 . Initial statement: feasible solutions $ND = 238$; Pareto optimal solutions $NP = 3$.

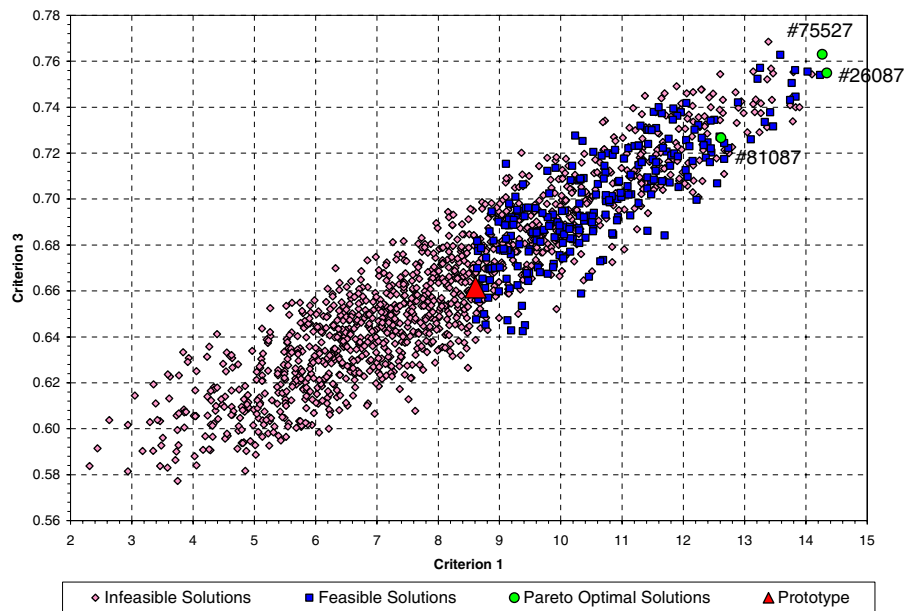


Figure 8. The dependency between criterion Φ_1 and pseudo-criterion Φ_3 .

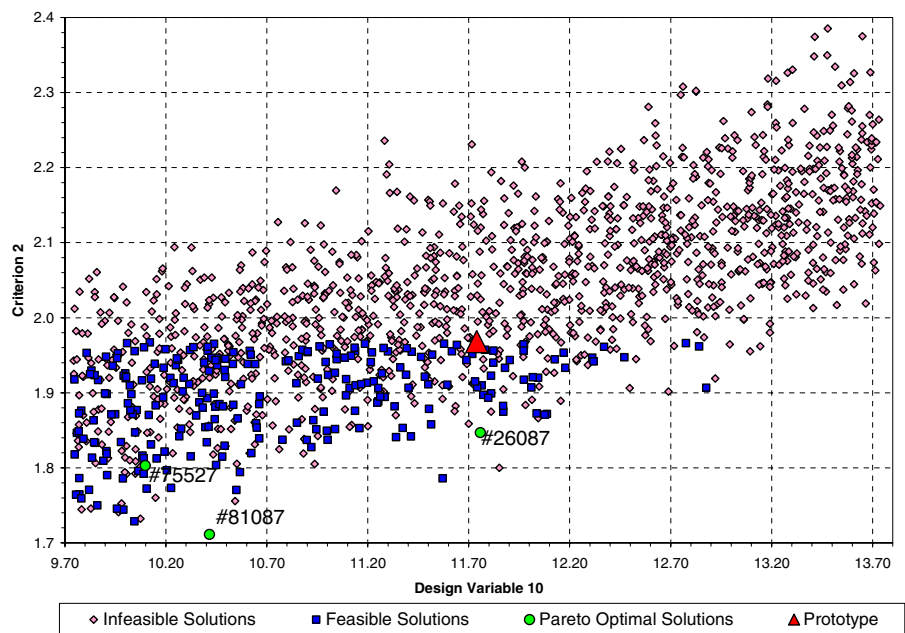


Figure 9. The dependency of criterion Φ_2 on design variable α_{10} .

After combining the feasible sets in both problem statements, we have constructed a combined Pareto set. No solution belonging to the initial statement

has entered into the combined Pareto set. In other words, all the results of optimization in the initial statement have been improved. The preference of the

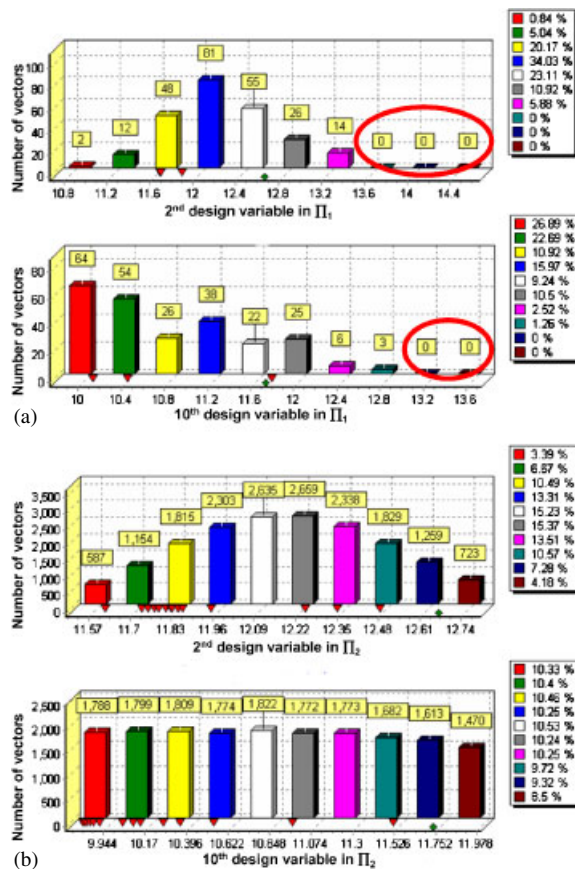


Figure 10. Histograms of the distribution of feasible and Pareto optimal solutions for the 2nd and 10th design variables: (a) corresponds to initial parallelepiped Π_1 and (b) corresponds to parallelepiped Π_2 . The percentage of designs entering the corresponding interval is indicated on the right of each histogram. The prototype is marked with a green diamond. The Pareto optimal vectors are marked with red triangles. The 'gaps' of the initial range for the 2nd and 10th design variables are circled.

expert was given to the vector # 74223. This vector surpassed all solutions in the initial statement by two performance criteria simultaneously, see Table II and Figure 12.

6.3. Final (third) statement and solution of the optimization problem. Parallelepiped Π_3

By analysing the feasible solutions from Π_2 , a new parallelepiped Π_3 was constructed similarly as in Section 6.2. After $N = 131\,072$ tests, $N_1 = 34\,986$ vectors entered the test table. Compared to the initial and second statements of the problem, all

criteria constraints now were determined on the basis of analysis of the test tables. In particular, more rigid criteria constraints ($\Phi_1^{**} = 14.342$ and $\Phi_2^{**} = 1.711$) were formulated. The pseudo-criteria constraints were also revised. As a results, we obtained $ND = 847$ feasible solutions and $NP = 7$ Pareto optimal solutions.

The smaller number of the feasible solutions compared to the previous problem statement can be explained by much stronger performance criteria constraints. The Pareto optimal solutions (# 113487, # 4145, # 68410, # 39801, # 53988, # 72461, # 75110) are shown in Figure 11. The values of criteria of these solutions are given in Table II.

The analysis of Pareto optimal solutions revealed that solution # 75110 surpassed seven solutions from the second statement (# 80159, # 87511, # 78907, # 31819, # 64407, # 63919, # 74223) by two criteria simultaneously. Solution # 113487 surpasses five solutions from the second statement (# 22671, # 90819, # 105823, # 116871, # 21855) by two criteria simultaneously. The expert's preference was given to solution # 113487.

Using the feasible solutions from the second and final statement, the combined Pareto set was constructed. The combined Pareto set includes all seven Pareto optimal solutions from the final statement and only two solutions # 106467, # 49109 belonging to the second statement, see Figure 12. Further attempts to improve the obtained solutions have not yielded any new interesting results.

The stability of the most interesting Pareto optimal solutions was investigated with respect to small variations of the parameters in the vicinity of these solutions. To this end, we constructed parallelepipeds centred in the Pareto optimal solutions and performed 1024 tests in each parallelepiped. The corresponding variations in the criteria were small and insignificant, which indicated the stability of the solutions.

The overall dynamics of improving a prototype on the basis of two corrections of the problem statement is shown in Figure 12.

In summary, the problem of improving a preliminary ship design prototype has been solved. This process included changing the range of variation of design variables and revising all criteria constraints. As a result of multicriteria analysis, the parallelepiped Π_2 was constructed. The volume of parallelepiped Π_2 was considerably decreased in comparison with the volume of

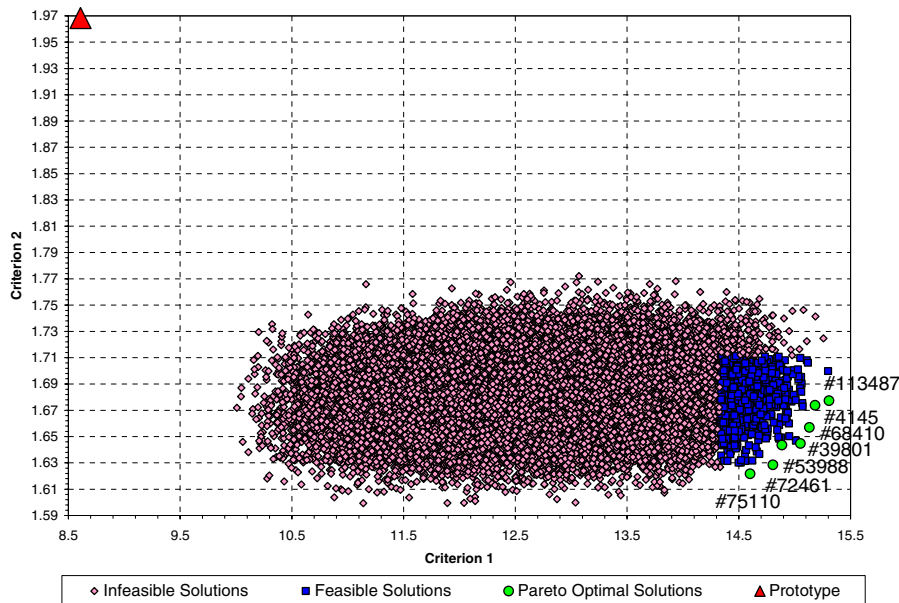


Figure 11. Dependency between criteria Φ_1 and Φ_2 . Final statement: feasible solutions ND = 847; Pareto optimal solutions NP = 7.

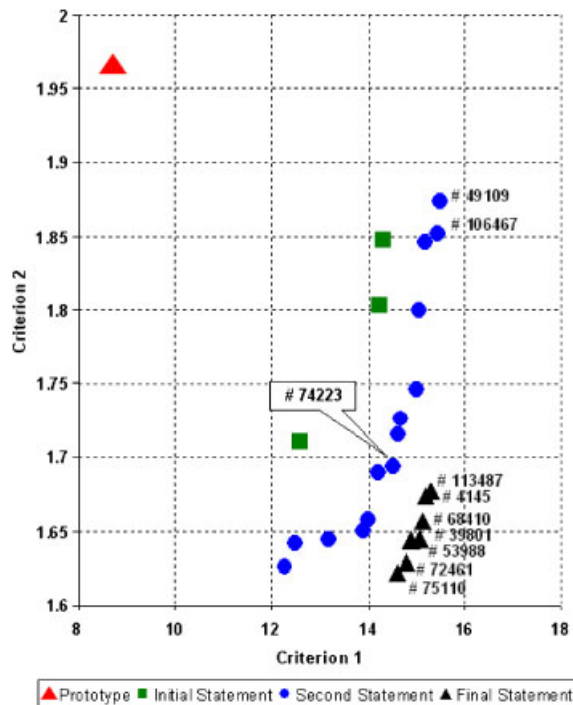


Figure 12. Pareto optimal solutions in the three statements and the combined Pareto optimal solutions.

parallelepiped Π_1 . Similarly, the parallelepiped Π_3 was constructed with a smaller volume than of parallelepiped Π_2 . This allowed a more careful and efficient investigation of the design variable and criteria spaces.

7. CONCLUSION

Construction of the feasible solution set is of fundamental importance in multicriteria real-life problems, especially when improving a prototype. The primary contribution of the present work is that on the basis of PSI method we provided various visualization and analysis techniques that facilitate construction of the feasible solution set and help solve the problem of improving a prototype. Furthermore, we described some basic features of this problem, provided a geometrical interpretation, and showed the necessity of constructing the combined Pareto set. Finally, we presented a case study where we aimed at improving a preliminary ship design prototype.

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